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Quantifying uncertainty in earthquake source parameters using the Large N LASSO Array: Collaborative Research with Massachusetts Institute of Technology and Boston University

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Abstract

We use a large dense array of seismometers to quantify spatial variation in site response, and its effects on estimates of earthquake stress drop, and also how source complexity leads to bias in spectral source modeling. Stress release during an earthquake is proportional to the slip divided by the length scale of the rupture. It is a basic property of earthquakes fundamental to understanding the physics of the source and its energy budget. The stress release also governs the amplitude of ground motions at the frequencies important for strong ground motion prediction and so is inherent to seismic hazard analysis. Many studies have attempted to characterize the high-frequency earthquake radiation spectrum and measured stress release using simple source models, to distinguish induced and tectonic earthquakes, and determine the factors controlling the earthquake rupture process. Small and moderate sized earthquakes are typically used for studies of source characterization because of their larger numbers, especially in regions of lower strain rate. Unfortunately, this work has led to inconclusive and controversial results, with different studies failing to agree within their calculated uncertainties. Ongoing analysis has found the major sources of error result from distinguishing source from path and site effects, and assuming simplistic source models, both a consequence of the limited data for most events, in terms of number of stations and frequency range of the signal.

Large-N arrays of seismometers provide an unprecedented opportunity to characterize smaller earthquake sources, using extremely well recorded wavefields without spatial aliasing, and quantify the real uncertainties from analyses using smaller station numbers and various methods. The LArge-n Seismic Survey in Oklahoma (LASSO), a USGS deployment in 2016, is the largest public archive to date (in terms of spatial coverage: over 1800 stations, and frequency range: 500 samples/s) to record the wavefield of smaller earthquakes.

We investigate the site response across the dense array, by calculating relative peak ground velocity from regional and teleseismic events in different frequency ranges. We find a strong correlation between the site amplification at high frequencies and the surficial geology. Sites with high amplification are typically located on young alluvial sedimentary deposits. At lower frequencies the PGV is more dependent on the earthquake radiation pattern.

We then use the site amplification effects and regression analysis to correct the measurements of seismic moment and corner frequency previously obtained using standard spectral fitting. We find that the estimated site effects decrease the spatial variability of the source parameters significantly.

Finally, we apply empirical Green's function methods in both the frequency and time domains to quantify the uncertainties in estimates of stress release and finite rupture extent of M2-3 earthquakes. We find that source complexity is a major cause of uncertainty, because it decreases the appropriateness of the simple source models in common use. We consider ways in which the variability resulting from source complexity can be accurately included in analysis of more typically-recorded events.

Project Significance

The joint goals of the proposed work are to quantify the uncertainties in estimates of small earthquake source dimension and stress release, and to develop improved approaches to characterize small earthquakes at higher frequencies. Whether earthquake sources are scale-invariant, and any dependence of earthquake source parameters on tectonic setting or other factors such as temperature, strain rate, of presence of fluids, are fundamental observations to understanding earthquake source physics and the factors that control rupture. They also directly impact predicted ground motion and hence seismic hazard prediction, as noted in the RFP Element III for EP and IS: "*Understanding earthquake phenomena and evaluating earthquake hazards requires research on the controlling processes and conditions, including anthropogenic influences*." Unfortunately, the uncertainties in most current estimates of small earthquake stress release and spectral shape are so large as to mask any reliable signal. For example, in a recent high-quality study of natural and induced earthquakes in different settings, Huang *et al*. (2017) found that none of the stress drop values or uncertainties that they calculated overlapped with those from an equally careful study by Boyd *et al*. (2017) for the same earthquakes, using similar approaches and data. To "*determine relations among fault properties, the dynamics of the earthquake source and ground motion*" we need more reliable, precise measurements of earthquake source parameters, and their real uncertainties. We use the unprecedented recordings of the wavefield of small earthquakes by the LArge-n Seismic Survey in Oklahoma (LASSO) to do just that. Since the earthquakes recorded by the LASSO array are likely to have been induced, we also apply "results from studies of earthquakes induced by anthropogenic activities to improve our understanding of natural earthquakes". The work forms part of a collaborative and integrated analysis using LASSO to investigate the earthquake source and our resolution of source parameters. Our work follows on from that of Kemna *et al*. (2020). Dr. Rebecca Harrington and Killian Kemna (both at Ruhr-Universität Bochum, Germany), and Dr. Elizabeth Cochran (USGS Pasasdena) provided us with the information we needed to build directly on their work, and in turn we are collaborating with Dr. Colin Pennington (USGS Moffett Field), to facilitate his finite-fault inversion analysis of the same earthquakes.

Introduction and Motivation

Since the pioneering work of Brune (1970) innumerable studies have attempted to use a relatively simple spectral method to estimate the source dimension and stress drop of small and moderate earthquakes in multiple settings (for example, Abercrombie 1995, Shearer *et al*., 2006, Huang *et al*., 2017, Boyd *et al*., 2017, and references therein). The goals of such studies have been to characterize the source process by estimating the spatial extent of slip and average stress drop. Together with the seismic moment, these parameters define the earthquake energy budget (Kanamori and Brodsky, 2004), and also the expected ground motions at frequencies of engineering interest (>1 Hz, Cotton *et al*., 2013, Trugman and Shearer, 2018). Different analyses have focused on determining whether earthquake sources are scale invariant (e.g. Abercrombie 1995, Abercrombie, 2013; Trugman and Shearer 2017), probing the energy budget (e.g. Abercrombie and Rice, 2005) and resolving whether the source process is controlled by factors such as temperature, strain rate, depth, or anthropogenic inducing (e.g. Viegas *et al.,* 2010, Allmann and Shearer, 2007; Sumy *et al.,* 2017; Boyd *et al.,* 2017, Huang *et al*., 2017, Trugman *et al*., 2017; Zhang *et al*., 2016). Whether induced earthquakes are different from natural ones, or whether earthquakes in different tectonic settings are different to one another has direct implications for seismic hazard prediction, particularly in regions of low seismicity.

Unfortunately, it has become clear that the uncertainties in these source dimension and stress drop estimates are large enough to significantly limit their interpretation (see review by Abercrombie, 2021). As the quantity and quality of data have increased for large earthquakes, they are typically modeled with more complex structural models, and source processes (e.g., Brown *et al.,* 2015 and references therein). For many earthquakes and regions of interest, there are too few good recordings to constrain such details and complexity, and so the challenge remains to obtain useful information from the available data.

An earthquake seismogram, *s(t)*, is the convolution of the radiation from the earthquake source, *e(t)*, with the combined propagation effects, *G(t)*, along the path, including both near-source and site effects, and finally the instrument response, *I(t)*;

$$s(t) = e(t) * G(t) * I(t). \qquad 1$$

If *I(t)* can be assumed known, then the problem is to separate *e(t)* and *G(t)* correctly, and this is the source of much of the uncertainty in stress drop measurements. In the frequency domain, body-wave spectra are often modeled assuming a simple circular source model and exponential attenuation

$$\dot{M}(f) = M_0 \left[\frac{e^{-\pi f t/Q}}{1+\left({}^{f}/_{f_c}\right)^{\gamma n}}\right]^{1/\gamma}, \qquad 2$$

where f is frequency, and f_c is the corner frequency, M_0 the seismic moment of the earthquake, n is the high-frequency fall off (n=2 is usual assumption), and γ is a constant controlling the shape of the corner; $\gamma = 1$ in the original Brune (1970) model and $\gamma=2$ in the sharper-cornered Boatwright (1980) version. The exponential term approximates the path effects over travel time t with quality factor Q. The finite, and often limited, frequency bandwidth of seismic recordings makes this separation harder.

Modeling recorded amplitude spectra with these simple source and attenuation models leads to large trade-offs between parameters (e.g., *Ko et al.,* 2012), and often underestimation of the stress drop (Kwiatek *et al*., 2014). Using a small, co-located earthquake as an empirical Green's function (EGF) to correct for all propagation effects is a relatively simple method of isolating the source process (e.g., Mori and Frankel, 1990). In spectral modeling, the ratio of the spectrum of a large earthquake, and a collocated smaller one, is fit using equation 2 for each so that the attenuation effects cancel, and the unknowns are the two corner frequencies and the ratio of seismic moments. This approach is an improvement on equation 1 in isolating the source (e.g., Kwiatek *et al*., 2014), but the selection of an appropriate EGF can lead to uncertainties (e.g., Kane *et al.,* 2011, 2013; Abercrombie, 2015 and Zhang *et al*., 2018). Also, Shearer *et al*. (2019) showed that the inversion fit is not always inadequately constrained. This approach cannot work at all for the smallest events in a data set, nor for events with no appropriate EGF. Shearer *et al*. (2006) developed a version of the method involving stacking large quantities of earthquakes over a range of magnitudes and using a global EGF function. Shearer *et al*. (2019) and Zhang *et al*. (2018) found that improvements are needed to correct for path effects more reliably, but the approach has promise for obtaining useful results for dense clusters of events.

Another problem with assuming a simple circular source model is that there is increasing evidence that small earthquake sources also involve directivity and variable slip and leading to complex source time functions as observed for larger events. For example, Folesky *et al*. (2016), Tomic *et al*. (2009) and Abercrombie *et al*. (2017) found clear evidence of directivity in well recorded small earthquakes, and Yamada *et al*. (2005), Wang *et al*. (2014), Uchide and Imanishi (2016), and Ruhl *et al*. (2017) all showed complexity and multiple subevents in similarly small events. Even if the earthquake ruptures a simple, symmetric, circular path of fault the corner frequency still varies with azimuth (Madariaga, 1976; Kaneko and Shearer, 2014), and increasingly so with small degrees of asymmetry (Kaneko and Shearer, 2015). This leads to the questions: what does a simple source model measure for such a complex rupture, what is the average stress drop (e.g., Brown *et al.,* 2015; Lin and Lapusta*,* 2018), and, perhaps most importantly, can estimates based on small numbers of stations that are inadequate to constrain azimuthal variation reveal any useful measurements?

Despite all this uncertainty, there is evidence of real systematic variation in earthquake sources. For example, Pennington *et al*. (2021a) calculated and compared source parameters estimated using multiple approaches for earthquakes in Prague, Ok. They found significant variability in absolute measurements between studies, but the relative values are surprisingly consistent over the different approaches.

The aim of our proposed work is to develop approaches to extract more reliable and better constrained measurements of earthquake stress release, and its variability from the large volumes of recordings of moderate and small earthquakes. Our work forms a step in a longer and more comprehensive analysis.

The LArge-n Seismic Survey in Oklahoma (LASSO)

To reveal the seismicity patterns, and detailed wavefield of induced earthquakes, the USGS deployed the temporary LASSO array of over 1800 nodal seismometers in 2016, Figure 1. Dougherty *et al*. (2016, 2019). The array, which lasted 34 days, was the first known academic dense deployment specifically designed to target induced seismicity in the region. It is significantly larger than other nodal deployments of a few hundred instruments for the observation of the induced seismicity (Sweet *et al*., 2018, Kim and Keranen, 2018).

The array extended 25 km by 32 km in northern OK (see Figure 1). The 10 Hz, single (vertical) component nodes were buried in shallow holes at a spacing of approximately 400 m, along county roads. The low traffic along the roads means that the sites were generally quiet. The data were recorded continuously at 500 samples/s, and are archived at the IRIS data center (Dougherty *et al.,* 2016, 2019).

The LASSO array detected the 112 events in the OGS catalog for the mapped area of Figure 1, including eight events $2.4 \leq M \leq 3$ that occurred within the footprint of the array, with over 180° of azimuthal coverage.

Dougherty *et al. (*2019) obtained a more detailed catalog of events recorded by LASSO. They used STA/LTA pick detections on at least 110 stations to make an initial catalog. They then used matched-filter detection to expand this catalog further resulting in a catalog of 3640 located events in and around the array (Dougherty *et al*., 2018, 2019). Most of the events are within the depth range 1-6 km, with a median depth of 3.6 km.

Kemna *et al*. (2020) performed an initial analysis of source parameters and uncertainty using recordings of the LASSO array. They compared analysis of the entire array using either spectral fitting, or empirical Green's analysis, with the variability of individual station measurements. They found that the precision of source properties estimated from direct phase arrivals for arrays with less than 20 stations should be assumed to be not less than 30% and could be as high as 150% if less than five stations are used. Our work follows on directly from this study.

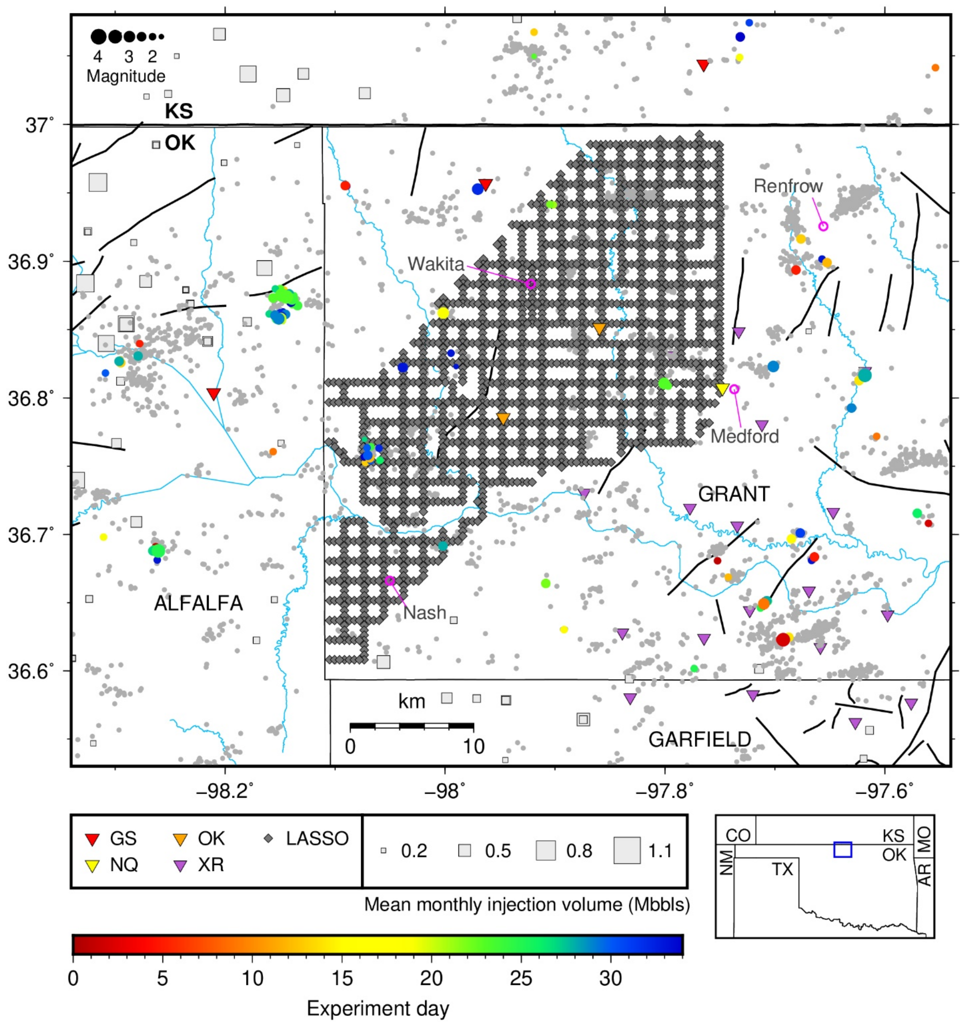

Figure 1. Map of the Large-n Seismic Survey in Oklahoma (LASSO) experiment study area in northern Oklahoma, after Dougherty et al., 2019 Nodal stations of the LASSO array (gray diamonds) and concurrently operating temporary and permanent broadband stations (inverted triangles, GS: USGS network, NQ: Netquakes, OK: Oklahoma Seismic Network, XR: seismicity near the Nemaha fault in northern Oklahoma) are shown. Seismicity from the Oklahoma Geological Survey (OGS) catalog and the ANSS (Advanced National Seismic System Comprehensive Earthquake Catalog which occurred during the LASSO array deployment is indicated by dots colored by day of experiment and scaled by magnitude. Seismicity from the same catalogs between 2015/01/01 and 2016/04/10 is shown as grey dots. The locations of high-rate wastewater injection wells are denoted by light grey squares, scaled by mean monthly injection volume. Thick black lines indicate mapped faults in Oklahoma, and thin black lines mark county borders in Oklahoma. Light blue lines mark streams. (Inset) Regional map of south central S showing location of study area (blue box) labeled with states.

Analysis of Spatial Variation in Site Effects:

We begin by quantifying the site response, not specifically considered by Kemna *et al.* (2020), and potentially responsible for some of the spatially coherent variability that they observed using spectral modeling. We use the recordings of distant earthquakes as these can be considered closest to plane waves arriving at the array having experienced common source and path effects. We use large (M>6) earthquakes from the Middle America subduction zone, which contain frequency information only below 1 Hz. These show consistent patterns most likely representing source focal

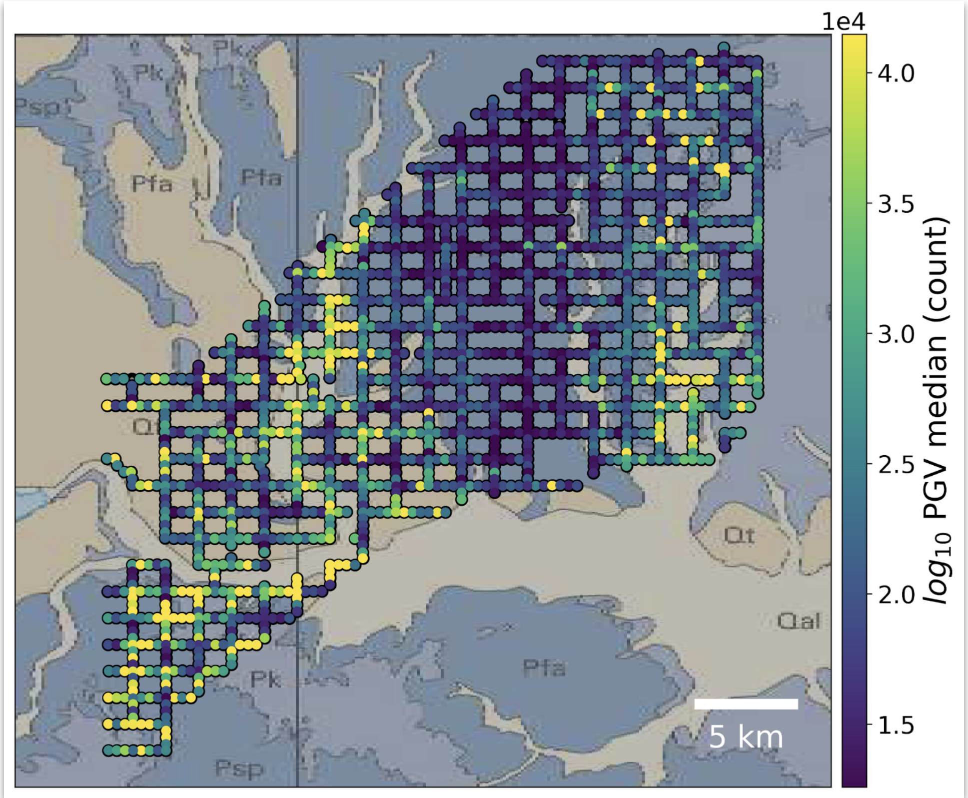

Figure 2: Comparison between Peak Ground Velocity (PGV) and surficial geology. We calculate the median PGV (10-15 Hz) from 14 regional earthquakes, showing clear spatial patterns, consistent between events. Surficial geology map (Heran et al., 2003) showing High PGV correlates with deposits of Quaternary alluvium (Qt, Qal) whereas PGV is lower in regions where Permian shale and sandstone are exposed (Pfa, Pk and PspP). The region of high PGV in the southwest corner correlates with a region noted to be very sandy during array deployment suggesting unmapped soils or other surficial deposits.

mechanism, but are too long wavelength to investigate shallow site response. To investigate shallower effects, we use LASSO recordings of M3-4 earthquakes in southern Oklahoma, 130-150 km away. We calculate the RMS amplitudes in moving windows, relative to the median for the array, and similarly the peak ground velocity (PGV), following Johnson *et al*. (2020). The results of the two methods are very consistent, and also stable from event to event. The stacked variation in PGV is shown in Figure 2. The site response measurements correlate well with the surficial geology map. We find that sites with high amplification are typically located on young alluvial sedimentary deposits. The area of high amplification in the south west does not correspond to a mapped deposit, but it is a region where very sandy soils and surface deposits were noticed during the LASSO deployment. We are consulting with the Oklahoma Geological Society to investigate this further.

Using Site Effects to correct source parameters from spectral fitting:

Kemna *et al*. (2020) found some distinct spatial variations in their estimates of corner frequency, seismic moment and stress drop from spectral modeling that did not consider site effects (Figure 3a). We find significant correlation between their source parameter estimates and our site response measurements (Figure 3b). We determine a simple relationship between the source parameter estimates and the relative PGV using linear and quadratic regression analyses. We use these to correct the source parameter estimates, finding a significant reduction in the spatial variability of the source parameters, particularly corner frequency (Figure 3c). The average RMS amplitude deviation among sites is about 50% of the fc deviation for one of the best recorded events. The seismic moments seem to be more affected by the radiation pattern.

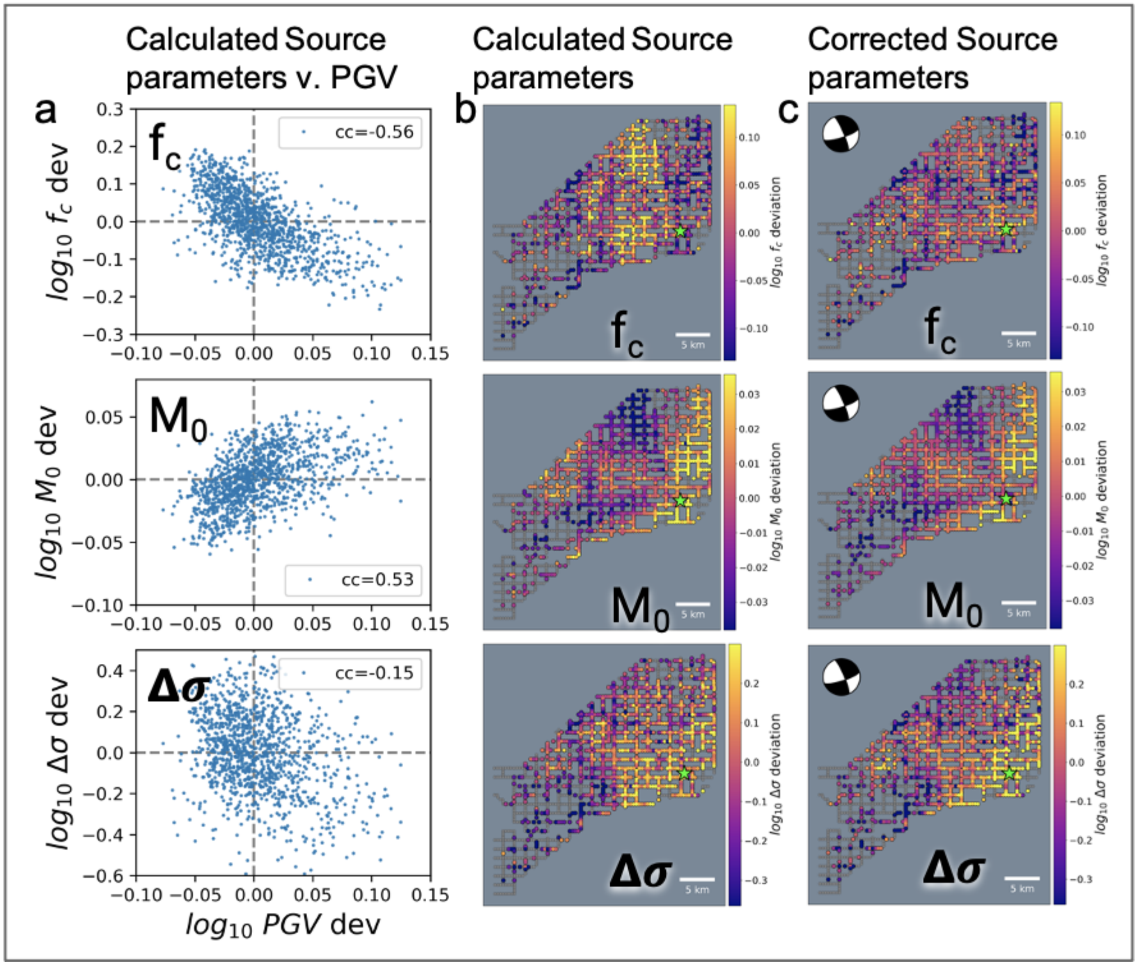

Figure 3: Comparison between Peak Ground Velocity (PGV) and results of spectral fitting for source parameters by Kemna et al. (2020). (a) Median PGV (10-15 Hz) from 14 regional earthquakes, plotted against previously determined source parameters showing significant correlation. (b) Spatial distribution of calculated source parameters shown in (a). (c) Source parameters after using their correlation with the PGV to correct for site effects. Note the lower variability and less clear spatial patterns. Especially for corner frequency which is strongly affected by amplification in the 10-15 Hz range.

Detailed EGF analysis of small earthquake sources:

We apply an Empirical Green's Function (EGF) method to estimate source parameters from spectral ratios and remove the influences of site amplification on corner frequency (Figure 4). To obtain more stable estimates of the source spectrum, and its spatial variability, we divide the array into smaller sub-arrays, which we stack individually. We experiment with different groupings of stations based on azimuth and distance and stack their spectral ratios and source time functions (STF) to estimate source parameters for some of the best recorded events (Mw 2.2~2.7) within the array. We vary the station density, azimuth, and distance range of the sub-groups and analyze the effects on the fitting results.

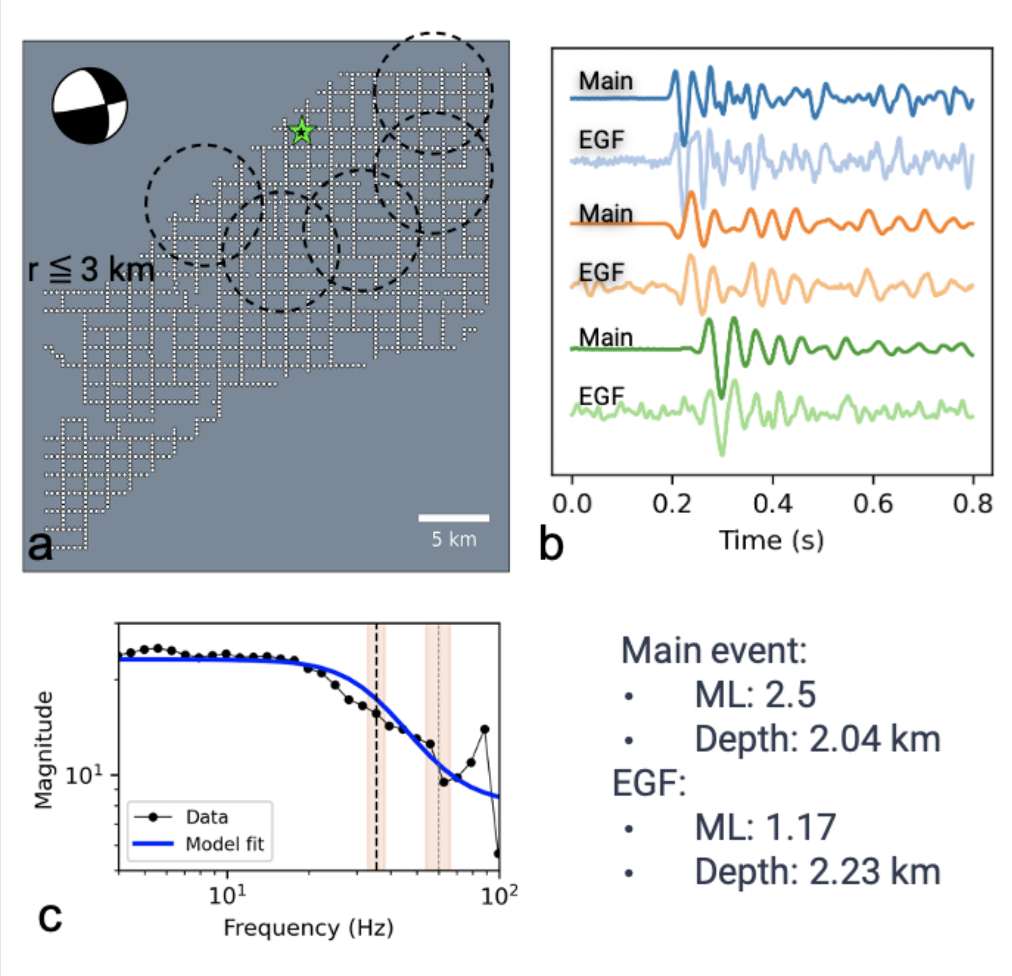

Figure 4: Empirical Green's function analysis of a ML2.5 earthquake using sub-arrays for stability. (a) Selection of subarrays, (b) example seismograms at individual stations, and (c) stacked spectral ratio, with model fit of a stacked sub-array. Vertical dashed lines and shaded regions indicate the two corner frequencies and their uncertainties, respectively.

The combination of time-domain and frequency-domain analyses provides useful insights. The STFs show gradual spatial variations that reflect the rupture directivity and also provide evidence of complex rupture in the form of distinct sub-events (Figure 5). The spectral ratios show considerable variation at high frequency consistent with the complex rupture, making simple modeling with a circular source model difficult (Figure 6). As our aim is to investigate the uncertainty in using simple spectral methods for relatively poorly-recorded events, we experiment with different constraints on the simple circular source model analysis. We are also collaborating with Dr. Colin Pennington (USGS) to obtain more detailed finite-fault models of the earthquakes best recorded by LASSO (Pennington *et al.*, 2021b). We find that it is necessary to constrain the second corner frequency in the modeling (see Shearer *et al.*, 2019), to prevent the source complexity leading to significant bias in the resulting corner frequency estimates of the larger events.

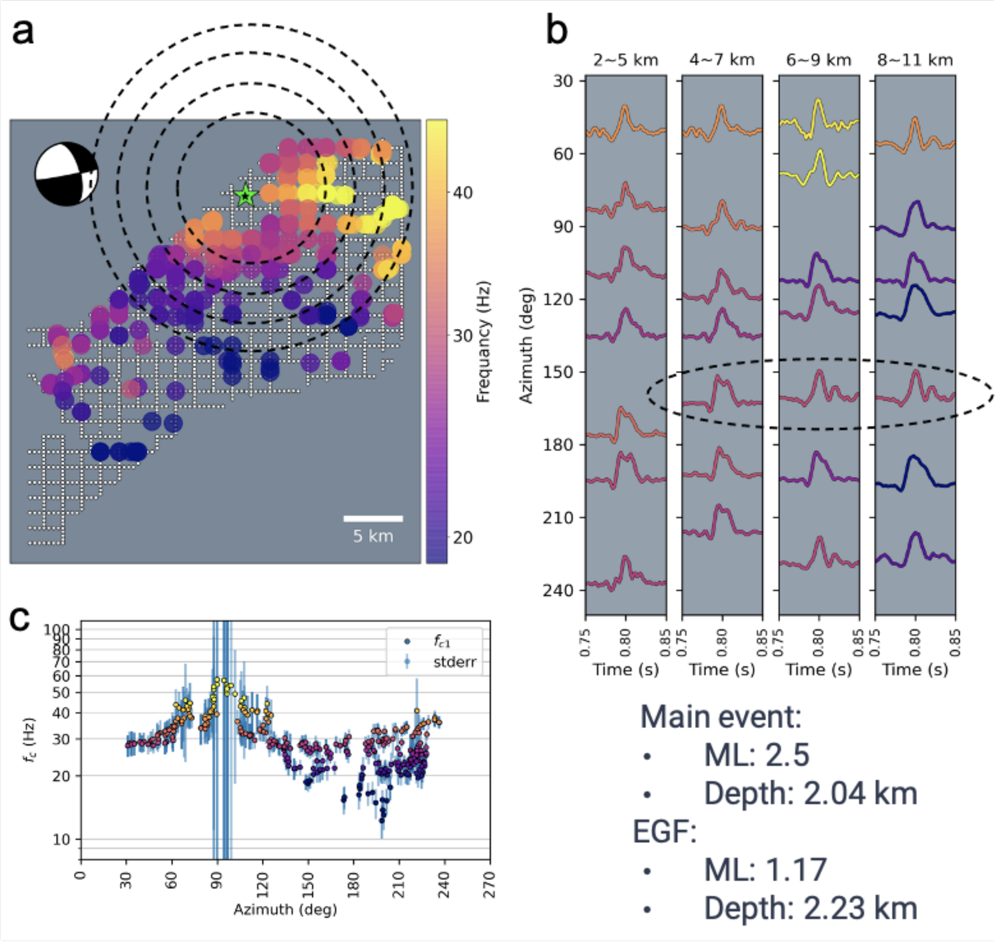

Figure 5: Spatial variation in corner frequency measurement, and source time functions indicates source complexity. (a) Spatial variation in corner frequency of larger event at centers of overlapping sub-arrays. (b) Source time functions at selected sub-arrays showing azimuthal and distance variation. Note the double pulses, for example, as indicated by the dashed ellipse, indicating multiple sub-events. (c) Modeled corner frequencies as a function of azimuth, with uncertainties.

Quantification of Uncertainties:

Having obtained the best estimates for source parameters, by including site effects in simple spectral modelling, and also performing EGF analysis of the larger events, we can quantify the variability that results from using smaller sub-groups of stations. We are doing this both using the simple spectral methods, and also by investigating the reliability of finite-fault inversion (Pennington *et al*., 2021b). We use a boot strapping approach, in which we randomly select several receivers (~5, 10 or 20 stations) from the entire array, to simulate a more typical seismology dataset and estimate earthquake source parameters. We also compare the use of sub-arrays versus single stations to look at azimuthal variability and directivity of the source. There are a lot of uncertainties, but also real source variability is clear. These results are already proving important to guiding related ongoing work into how best to obtain useful and reliable quantification of the high frequency radiation of small, poorly-recorded, complex earthquakes.

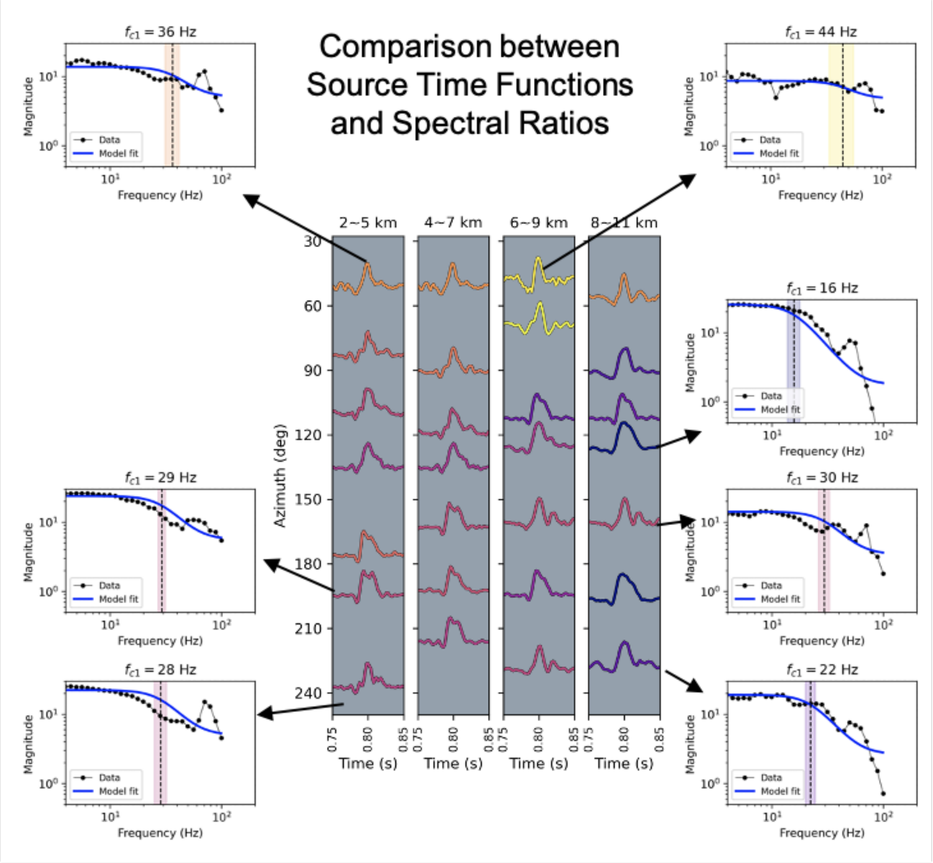

Figure 6: Comparison of spectral ratios and source time functions. Note how in the spectral ratios, the high frequency amplitudes, corner frequency, and amplitude and frequency range of the “bump” at around 50-70 Hz vary with duration, and simplicity of the source time functions. The complex nature of this event (multiple subevents) means it is not well fit by a simple circular model. The simple single corner frequency measurement is therefore not a good measure of the stress drop, or the high frequency radiated energy.

Data Management Plan

The proposed work uses previously collected seismic data from the deployment of the USGS LASSO array. These data are all archived at the IRIS Data Management Center from where we download them (Dougherty *et al*., 2016, 2019).

Our analysis has produced new measurements and source parameter results. The various parameters needed to reproduce these results (including, but not limited to, catalogues of stations, events, time windows, frequency ranges) together with the resulting measurements will be archived in the electronic supplements to our planned peer-reviewed publication, or in an associated MIT-based data repository. We have already shared these parameters with Dr. Colin Pennington (USGS) to enable him to make rapid progress on his work.

Publications and Dissemination

We are presenting the results of this analysis in two presentations at the 2021 Annual Meeting of the Seismological Society of America (Chang *et al*., 2021; and Pennington *et al*., 2021b). We are writing a manuscript for submission to peer-reviewed journal.

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