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A Hierarchical Bayes Inversion Approach for Site Characterization Using Surface Wave Measurements

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Abstract

In this project, a hierarchical Bayesian model inversion method is developed for site characterization and site response prediction using surface wave measurements and H/V spectral ratios. The hierarchical Bayesian method estimates a velocity profile with uncertainty quantification using both surface wave derived dispersion curves and ambient noise derived H/V spectral ratios. In the proposed hierarchical inversion framework, prior distributions are assumed for the updating parameters which are shear wave velocity in each soil layer. The prior probability distributions of shear wave velocities are updated to their posterior distributions through the hierarchical Bayes inference where the mean and covariance of updating parameters are estimated as hyperparameters. The Metropolis-Hastings algorithm and Gibbs sampler are used for estimating the updating parameters and hyperparameters. The framework is initially evaluated numerically when applied to a model of a 4-layer soil profile. Accuracy of the proposed inversion process is compared with the classical Bayesian inversion framework for different amounts of available data and the effect of adding fundamental frequency terms from H/V is investigated. The updated soil profiles are then used to estimate the site response transfer function and its uncertainty bounds. The proposed inversion process is evaluated experimentally using field measurements at a site in the greater Boston area. The hierarchical Bayesian inference approach provides a mechanism to use both the dispersion curve and the H/V spectral ratio in the shear wave velocity inversion. The estimated uncertainty of model parameters and modeling errors are used to provide uncertainty bounds for the predicted site response transfer function. The estimated uncertainty bounds derived from the hierarchical Bayesian approach using dispersion curves and H/V spectral ratios are more realistic than those using classical Bayesian inference schemes.

1. Introduction

Non-invasive site characterization techniques such as, Spectral Analysis of Surface Waves (SASW) tests and horizontal-to-vertical spectral ratio (HVSR) microtremor methods, have gained more attention for use in site response prediction in the last two decades. The main advantage of these methods compared to invasive methods like down-hole and cross-hole methods is their lower cost which is achieved by eliminating the dependency on boreholes needed for invasive methods (Nazarian et al., 1983; Park et al., 1999; Bonnefoy-Claudet et al., 2006). But one of the biggest challenges in using these approaches in site response prediction is how to characterize the uncertainty so that it can be carried through to the site response transfer function.

HVSR has been used to characterize the resonance frequency directly for site response studies (Hassani and Atkinson, 2016) but is not typically linked to estimation of velocity profiles. However, because HVSR is known to accurately estimate the resonant frequency of a site, it provides information on the characterization of soil layers and can help to constrain the inversion process. Both dispersion curves from the SASW test and the transfer functions from the HVSR microtremor data has been used for the inversion process (Xia et al. 1999, Arai et al. 2005).

Estimating the uncertainty of the shear velocity profile is an important step in developing a site response transfer function (Toro, 1995; Lai et al., 2005; Moss, 2008; Griffiths et al., 2016; Teague et al., 2018). Recent improvements such as those recommended by Griffiths et al. (2016) and Teague et al. (2018) make connections between uncertainty in field measurements and the propagation of uncertainty in site response transfer functions. These methods do not use Bayesian inference.

In this project, a hierarchical Bayesian inference method is developed for site characterization. In this approach, data from both SASW dispersion curves and HVSR estimates of fundamental period are combined to provide a more accurate estimate of velocity profiles with an explicit estimate of uncertainty. The hierarchical Bayesian inference method provides a realistic estimate of uncertainty in shear wave velocity for a given profile by accounting for modeling errors, measurement noise, and inherent variability in model parameters (shear wave velocity in a soil layer). The estimated uncertainty can then be propagated to the site response transfer function.

2. Model Inversion from Surface Wave Measurements

2.1. Classical Bayesian Inference

Bayesian inference which has been used in many inversion and model updating applications in recent studies is based on the Bayes theorem:

$$p(\boldsymbol{\theta} \mid \mathbf{D}) = c p(\mathbf{D} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta}) \tag{1}$$

where $p(\boldsymbol{\theta}|\mathbf{D})$ is the posterior probability distribution of updating parameter $\boldsymbol{\theta}$ given the measured data $\mathbf{D}$, $p(\boldsymbol{\theta})$ is the prior distribution representing our prior understanding of the updating parameters, $p(\mathbf{D}|\boldsymbol{\theta})$ is the likelihood function reflecting the information from measured data, and c is a constant value that ensures the left-hand side of the equation will integrate to 1 and is equal to $1/p(\mathbf{D})$ with $p(\mathbf{D})$ being the 'evidence'.

In order to formulate the Bayesian inversion framework, we need to choose the updating parameters of interest and define the error functions for the problem. In this project, we assume we know the soil profile (number and thickness of layers) and shear wave velocities of different soil layers are the updating parameters $\boldsymbol{\theta}$. Two sets of error functions are defined based on the available data:

$$e_f^{(t)} = \frac{\tilde{f}^{(t)} - f(\boldsymbol{\theta}^{(t)})}{\tilde{f}^{(t)}} \tag{2}$$

$$\mathbf{e}_{\mathrm{V}}^{(t)} = \frac{\tilde{\mathbf{V}}^{(t)} - \mathbf{V}(\boldsymbol{\theta}^{(t)})}{N_d \bar{\tilde{\mathbf{V}}}^{(t)}} \tag{3}$$

In these equations, e_f is the fundamental frequency error function defined as the difference between measured fundamental frequency $\tilde{f}$ from HVSR data and the calculated fundamental frequency from the model $f(\boldsymbol{\theta})$. The phase velocity error function $\mathbf{e}_{\mathrm{v}}$ is defined as the normalized difference between phase velocities of measured dispersion curves from the data $\tilde{\mathbf{V}}$, and the phase velocities of calculated dispersion curves from the model $\mathbf{V}(\boldsymbol{\theta})$. Superscript t stands for the dataset number and N_d represents the number of points extracted from the dispersion curve. It is worth noting that the hierarchical Bayes approach is best suited when several datasets are available. Values of $f(\boldsymbol{\theta})$ and $\mathbf{V}(\boldsymbol{\theta})$ are calculated based on soil models developed using Geopsy software package (Wathelet et al. 2020). These error functions are assumed to follow zero mean Gaussian distributions:

$$e_f \sim N(0, \sigma_f^2) \tag{4}$$

$$\mathbf{e}_{\mathrm{V}} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\mathrm{V}}) \tag{5}$$

where σ_f^2 and $\boldsymbol{\Sigma}_{\mathrm{V}}$ are the variance of the fundamental frequency error function, and the covariance matrix of the phase velocity error function, respectively.

Assuming N_t statistically independent sets of data, the likelihood function of the problem can be expanded as:

$$p(\tilde{f}^{(1)} \ldots \tilde{f}^{(N_t)}, \tilde{\mathbf{V}}^{(1)} \ldots \tilde{\mathbf{V}}^{(N_t)} \mid \boldsymbol{\theta}) = \prod_{t=1}^{N_t} N(\tilde{f}^{(t)} \mid f(\boldsymbol{\theta}), \sigma_f^2) N(\tilde{\mathbf{V}}^{(t)} \mid \mathbf{V}(\boldsymbol{\theta}), \boldsymbol{\Sigma}_{\mathrm{V}}) \tag{6}$$

Gaussian distributions with relatively large standard deviations are considered for the prior terms, effectively making less informed prior assumption about shear wave velocity of the layers. Metropolis algorithm (Metropolis et al., 1953) which is a Markov chain Monte Carlo (MCMC) method is used for numerical sampling of the posterior distributions for the classical Bayesian formulation of this section.

2.2. Hierarchical Bayesian inference

Studies have shown that while classical Bayesian inversion method can estimate the updating parameters with good accuracy, it consistently underestimates the uncertainties associated with the updating parameters (Sohn and Law 1997, Behmanesh and Moaveni 2015). Uncertainties derived from the method can only capture the uncertainty associated with the estimation process and therefore will generally decrease with adding more datasets until they go to zero. This is especially troubling when large number of measured data points are available which will effectively make the estimated uncertainties useless (since they will be unrealistically close to zero). One way to address this issue is to consider a distribution for the updating parameters and update the statistics of this distribution instead of directly updating the parameters (Behmanesh et al. 2015, Song et al. 2020). Similar idea can be used when dealing with the error function to avoid unrealistically small covariance estimates.

In the hierarchical Bayesian inversion method used in this study, Gaussian distribution is chosen to model the updating parameter and the error function:

$$\boldsymbol{\theta} \sim N(\boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}) \tag{7}$$

$$\mathbf{e}^{(t)} = \begin{bmatrix} e_f^{(t)} \\ \mathbf{e}_{\mathbf{V}}^{(t)} \end{bmatrix} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{e}}) \tag{8}$$

where $\boldsymbol{\mu}_{\boldsymbol{\theta}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ are mean and covariance of the updating parameters and $\boldsymbol{\Sigma}_{\mathbf{e}}$ is the covariance matrix for the error function. The error functions are assumed to be unbiased in this project to simplify the numerical application of this framework. Alternatively, bias can be estimated as the mean of the error function in a two-step approach (Song et al. 2019). Given these new hyperparameters, posterior probability distribution of updating parameters for each dataset t and the hyperparameters can be expressed as:

$$p(\boldsymbol{\theta}^{(t)}, \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\mathbf{e}} \mid \tilde{f}^{(t)}, \tilde{\mathbf{V}}^{(t)}) \propto p(\tilde{f}^{(t)}, \tilde{\mathbf{V}}^{(t)} \mid \boldsymbol{\theta}^{(t)}, \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\mathbf{e}}) p(\boldsymbol{\theta}^{(t)}, \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\mathbf{e}}) \tag{9}$$

Considering that identified data features are not directly dependent on $\boldsymbol{\mu}_{\boldsymbol{\theta}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ while the updating parameters are only dependent on $\boldsymbol{\mu}_{\boldsymbol{\theta}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ and considering that the hyperparameters are independent from each other, equation (9) can be simplified as:

$$p(\boldsymbol{\theta}^{(t)}, \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\mathbf{e}} \mid \tilde{f}^{(t)}, \tilde{\mathbf{V}}^{(t)}) \propto p(\tilde{f}^{(t)}, \tilde{\mathbf{V}}^{(t)} \mid \boldsymbol{\theta}^{(t)}, \boldsymbol{\Sigma}_{\mathbf{e}}) p(\boldsymbol{\theta}^{(t)} \mid \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}) p(\boldsymbol{\mu}_{\boldsymbol{\theta}}) p(\boldsymbol{\Sigma}_{\boldsymbol{\theta}}) p(\boldsymbol{\Sigma}_{\mathbf{e}}) \tag{10}$$

To further simplify the formulation of posterior distribution, conjugate priors are used for $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ and $\boldsymbol{\Sigma}_{\mathbf{e}}$ and uniform distribution is assumed for $\boldsymbol{\mu}_{\boldsymbol{\theta}}$. Metropolis-Hasting algorithm is used within a Gibbs sampler for the numerical estimation of the posterior distribution (Song et al. 2020).

3. Numerical Case Study: Site with 4-Layer Soil Profile

3.1. Soil Model and Simulated Data

A numerical model of soil profile with 4 layers is considered in order to investigate the efficacy of the proposed method (Figure 1). Each layer is assumed to have equal depth of 20 meters, mass density of 1900 kg/m^3 and Poisson's ratio of 0.3. Shear wave velocity of the layers are assumed to have Gaussian distributions with mean values ranging from 191.0 m/s for the top layer to 454.1 m/s for the bottom layer. Standard deviations of the soil velocity distribution are also assumed to have a generally increasing pattern ranging from 10.6 and 9.9 m/s for the top layers to 32.3 m/s for the bottom layer. Assumed mean and standard deviation of the shear wave velocities are reported in Figure 1. The underlying bedrock is assumed to have a mass density of 2800 kg/m^3 and shear wave velocity of 1800 m/s.

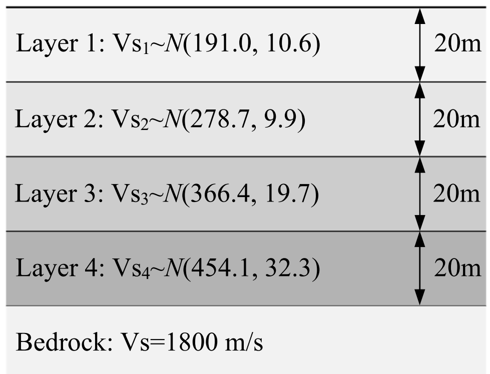

Figure 1: 4-Layer soil profile considered for the numerical study

A set of 100 dispersion curves and fundamental frequencies are simulated based on the considered soil profile. To capture the uncertainties associated with the measurements, these simulations are conducted by randomly sampling the shear wave velocity distributions. Simulated data is divided to subsets of N_t = 5, 10, 20, 50 and 100 pairs of dispersion curve and fundamental frequency. Classical and Hierarchical Bayesian inversion are used for different subsets of data to investigate the performance of each method in calculating the maximum a posteriori (MAP) estimate and the uncertainties associated with the shear wave velocities.

3.2. Inversion Results

In the first part of this numerical study, only dispersion curves are considered as measurements. To investigate the performance of the classical Bayesian inversion method in estimating the shear wave velocities and their associated uncertainties, this method is applied to different subsets of simulated dispersion curves. Figure 2 demonstrates the results considering 5, 50 and 100 sets of data. In this figure, the gray solid lines represent the actual assumed distributions of the shear wave velocities. It was observed that increasing the number of datasets decrease the estimated variability of the parameters and this decline was especially significant for the first layer where the estimated uncertainty gets close to zero for larger datasets.

A similar process was repeated for the hierarchical Bayesian framework assuming 5, 10 and 20 sets of data. As demonstrated in Figure 3, unlike the previous case, the estimated posterior probabilities for shear wave velocities are in good agreement with the actual assumed probability distributions and the accuracy of these estimates increases with the increasing datasets. It can be seen that this method captures the full shear wave velocity distribution in terms of central tendency and spread. This can be seen even for small subsets of data. However, for small subsets of data, MAP estimates are prone to bias caused by bias in selected datasets.

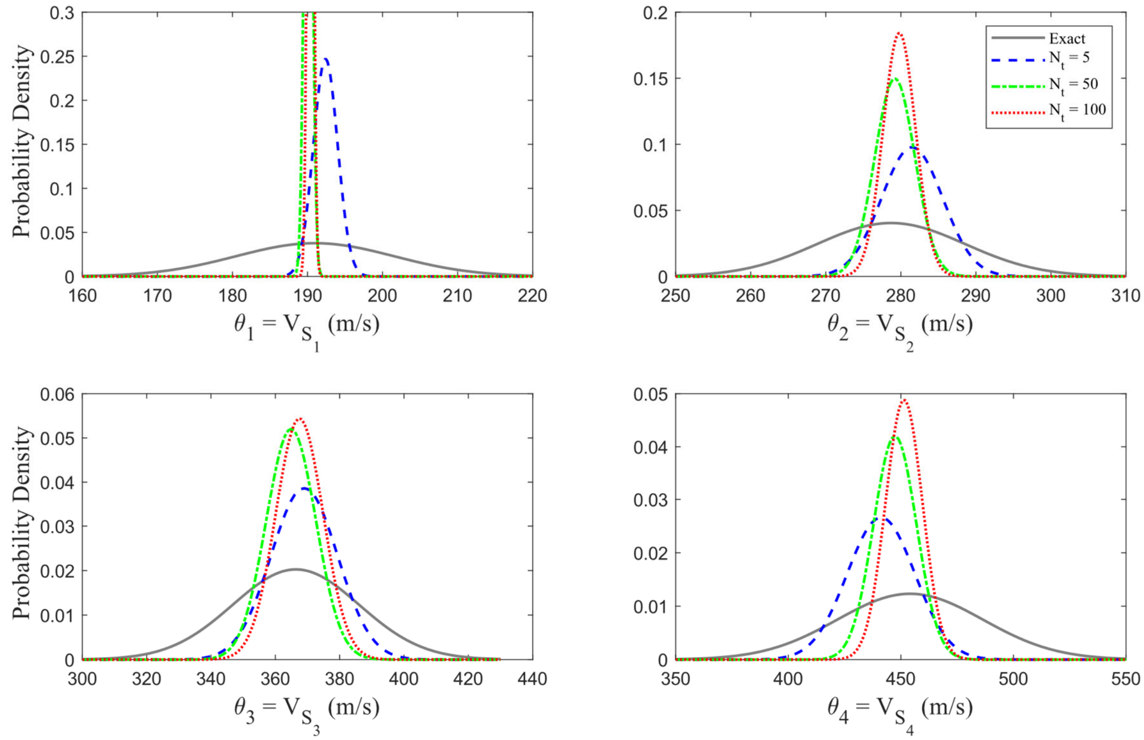

Figure 2: Posterior distributions of shear wave velocities based on classical Bayesian inversion process for different subsets of data

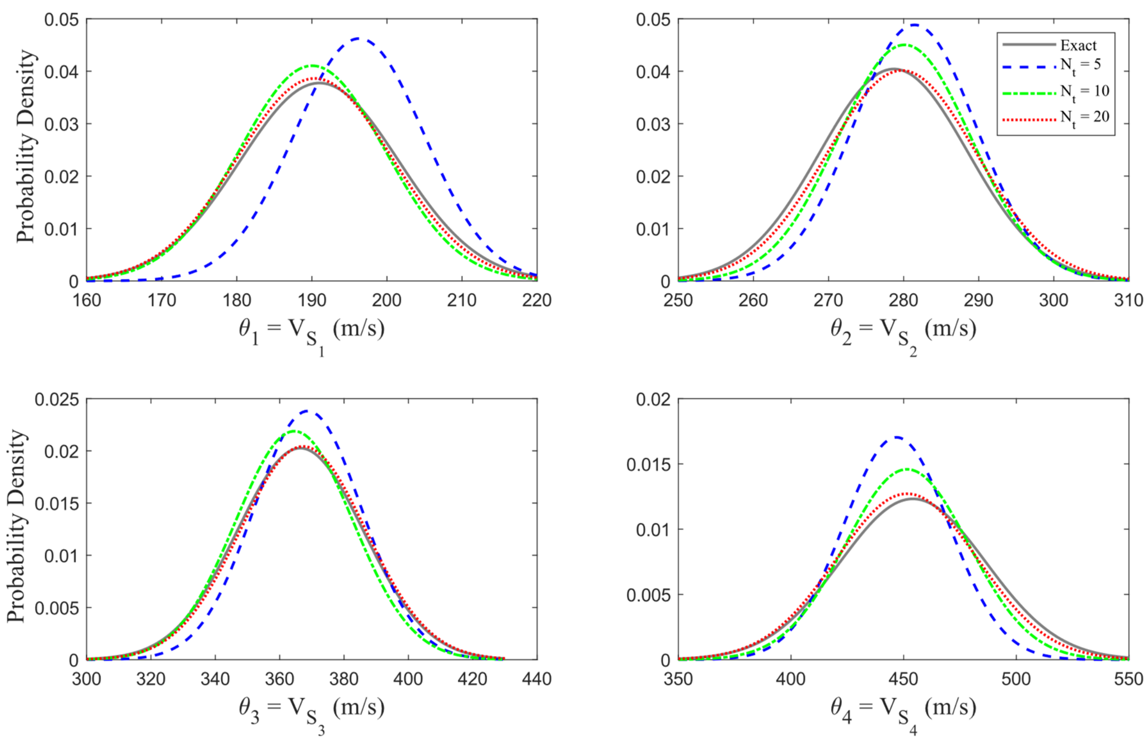

Figure 3: Posterior distributions of shear wave velocities based on hierarchical Bayesian inversion for different subsets of data

As previously mentioned, in the hierarchical Bayesian inversion methods the hyperparameters that characterize the distribution of updating parameters are estimated. Probability distribution for the estimated mean and covariance (hyperparameters) of the updating parameters are shown in Figure 4 and Figure 5. It was observed that the accuracy of the mean value estimates increases with increasing number of datasets while some bias is observed for the first layer when only 5 sets of data is used.

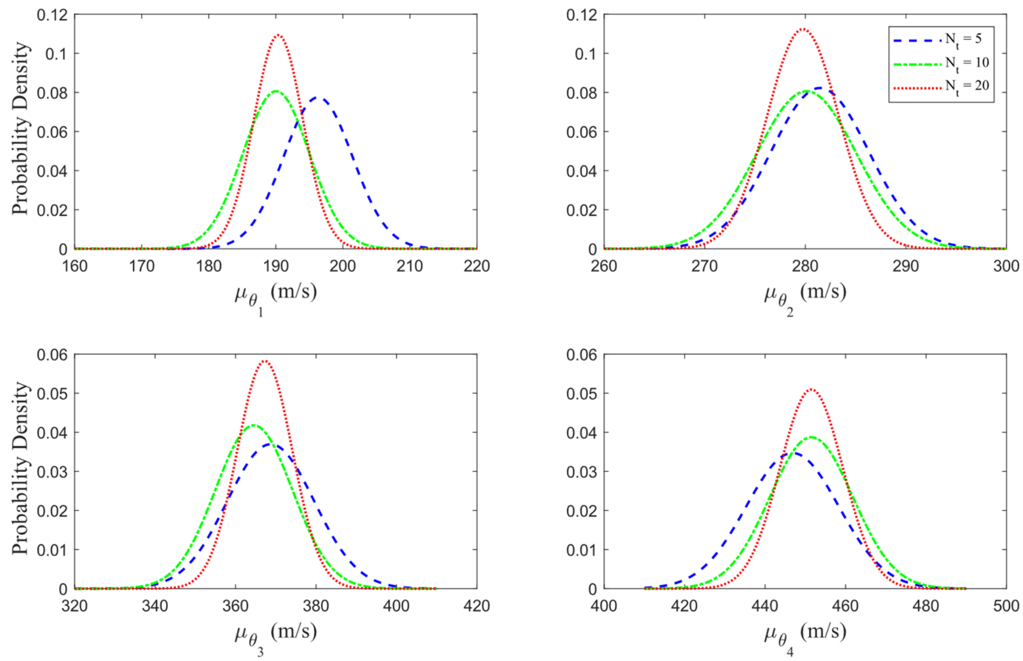

Figure 4: Probability distribution for mean shear wave velocities for different subsets of data

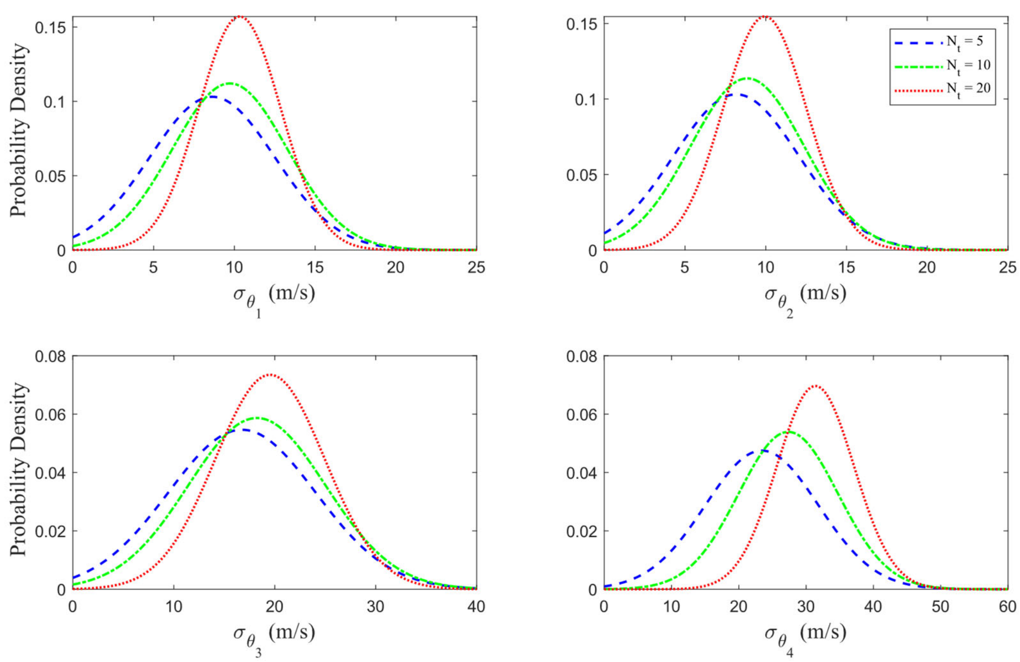

Figure 5: Probability distributions for standard deviations of shear wave velocities for different subsets of data

In the second part of this numerical study, we have added the simulated fundamental frequencies to the measured dispersion curve datasets in order to study the effect of combining surface-wave methods and HVSR to the inversion process. A subset of 10 dispersion curves is studied with and without addition of 100 simulated fundamental frequency values. Figure 6 demonstrates the posterior distribution of shear wave velocities for the two cases. Small difference is observed between the two sets of posterior distributions given that the 10 dispersion curves already characterized the distributions fairly accurately.

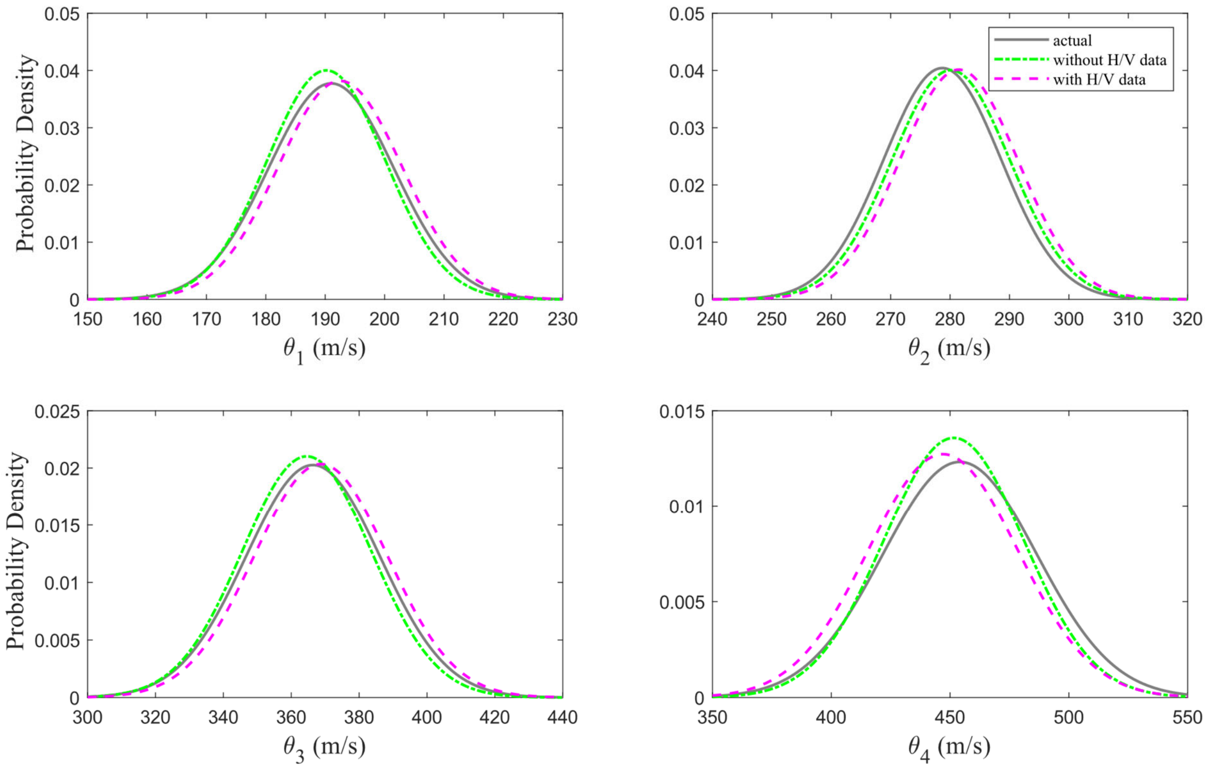

Figure 6: Posterior distributions of shear wave velocities based on hierarchical Bayesian inversion process with and without considering the frequency error

To further study the effect of adding fundamental frequencies to the inversion process, we investigated the accuracy of hyperparameters in the hierarchical framework. Figure 7 and Figure 8 show the distributions for mean and standard deviation of the updating parameters. As it can be seen in these figures, while the MAP value of the estimates do not show a significant change for the two cases (as was expected given the close match between the probability distributions of Figure 6), the standard deviations of the hyperparameters decrease with the addition of the fundamental frequency data. Therefore, while addition of fundamental frequencies did not improve our MAP estimates for $\boldsymbol{\mu}_{\boldsymbol{\theta}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ significantly, it decreased the uncertainties associated with the MAP estimates.

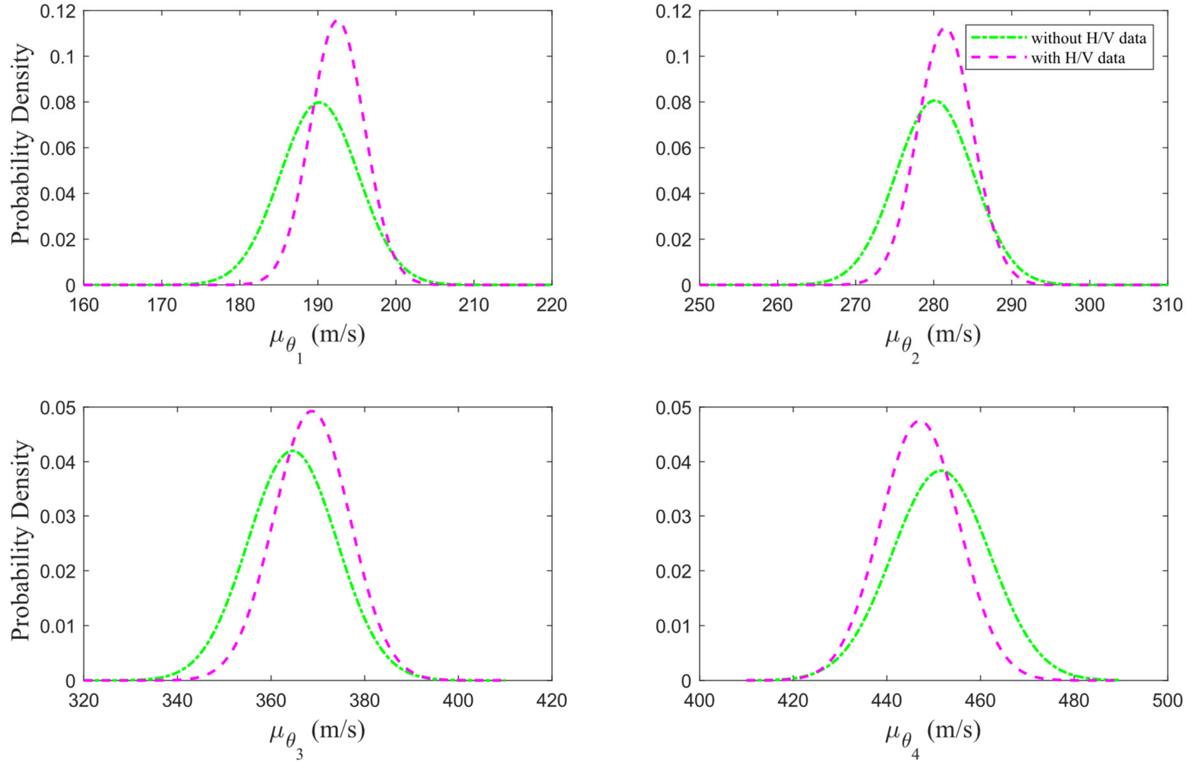

Figure 7: Probability distribution for mean shear wave velocities based on hierarchical Bayesian inversion process with and without considering the frequency error

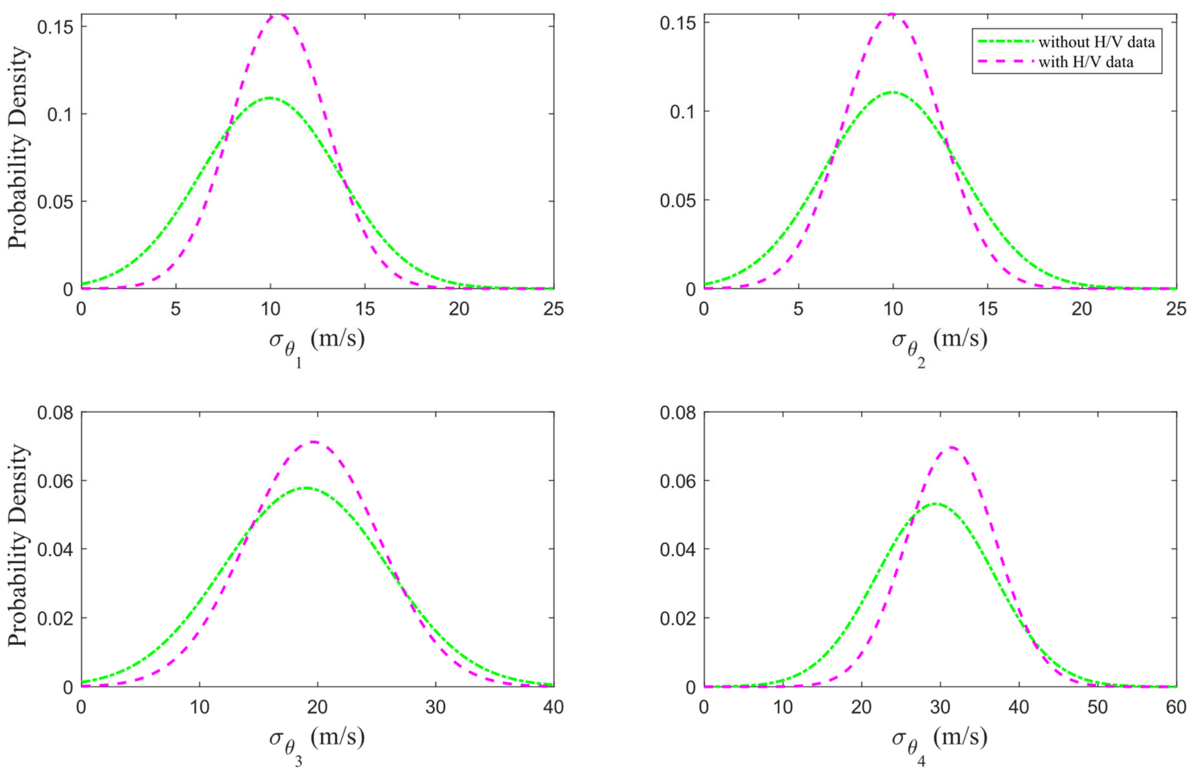

Figure 8: Probability distributions for standard deviations of shear wave velocities based on hierarchical Bayesian inversion process with and without considering the frequency error

3.3. Site Response Prediction

Based on the updated shear wave velocity distributions, we can estimate the transfer function for the soil profile. To do so, estimated shear wave velocity distributions are randomly sampled, and transfer functions are calculated for each set of randomly selected shear wave velocities. These transfer functions are plotted side by side and the 95 percent confidence intervals are calculated for different frequencies to demonstrate the actual and predicted site response and their uncertainties. Note that the nominal shear wave velocities were assumed to follow Gaussian distributions and therefore the actual site response is also presented with confidence bounds. These transfer functions are plotted and compared in Figure 9. It can be seen that there is a good match between the predicted and nominal site responses while the predicted response slightly underestimates the variability.

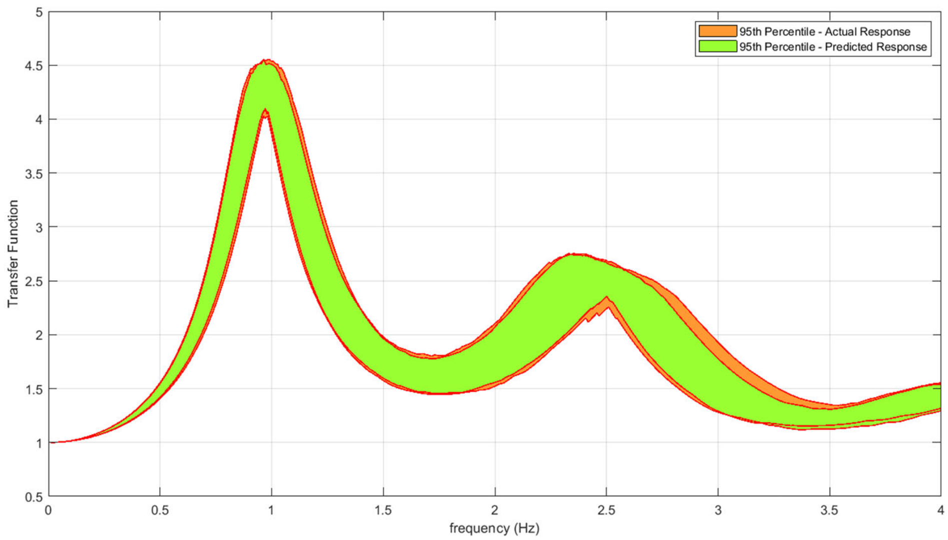

Figure 9: Transfer function for the 4-layer soil model considering the actual and the estimated shear wave velocity distributions

To further investigate the estimated site response, we focused on the estimated fundamental frequency. Figure 10a shows the probability distribution of the fundamental frequency (frequency of the peak) based on the actual and estimated shear wave velocity distributions while Figure 10b shows the probability distribution for the transfer function amplitude (amplitude of the peak). As it can be seen in both figures, there is a close agreement between the predicted (blue plots) and the nominal values (black plots). Similar to Figure 9, uncertainties associated with the predictions are slightly underestimated. Hierarchical Bayesian inversion framework can help us to further improve our results for the fundamental frequency distribution by providing estimates for the modeling errors. Given the definition of the frequency error function in Equation (2), we can include the effect of error values in our estimates:

$$f_{estimate} = \frac{f(\theta)}{1-e_f} \tag{11}$$

The error values estimated from the hierarchical Bayesian framework are randomly sampled in order to calculate the denominator of the right-hand side of Equation (11) while the frequency values in the numerator are also generated randomly. Results are shown as the red plot in Figure 10a from which it is evident that adding the modeling error improves the probabilistic predictions.

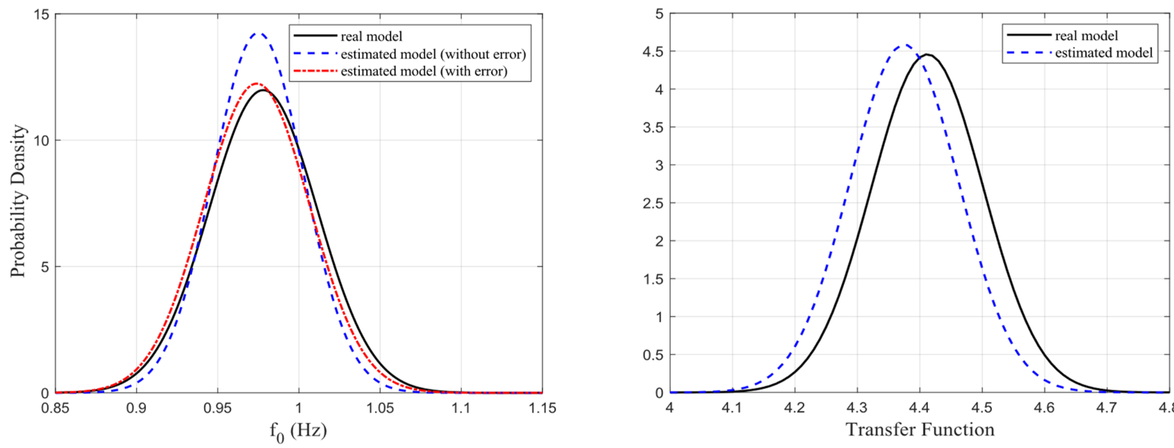

Figure 10: Probability distribution of (a) fundamental frequency and (b) peak amplification factor value for the actual and the estimated shear wave velocity distributions

4. Experimental Case Study: Danehy Park Site

The proposed inversion method is studied using the experimental data from a site in Cambridge, Massachusetts on the shore of the Charles River. The typical soil stratigraphy of this area consists of hard bedrock overlaid by layers of glacial till, Boston Blue Clay, sand and gravel, and artificial fill. This area is well studied and has been demonstrated to have a high impedance contrast leading to significant site amplification during earthquake shaking (Baise et al., 2016).

4.1. Available Data

Surface wave measurements collected at Danehy park, a sports field and dog park in Cambridge, MA north of Boston is used in this study. The site was classified as "fill over fluvial" with an average shear wave velocity of around 190 m/s in the previous study (Thompson et al. 2014). Dispersion curves were extracted from the SASW test data previously collected by Thompson et al. (2014) while HVSR were calculated based on ambient vibration data collected by the project team. Figure 11 shows the dispersion curve and HVSR for the collected data.

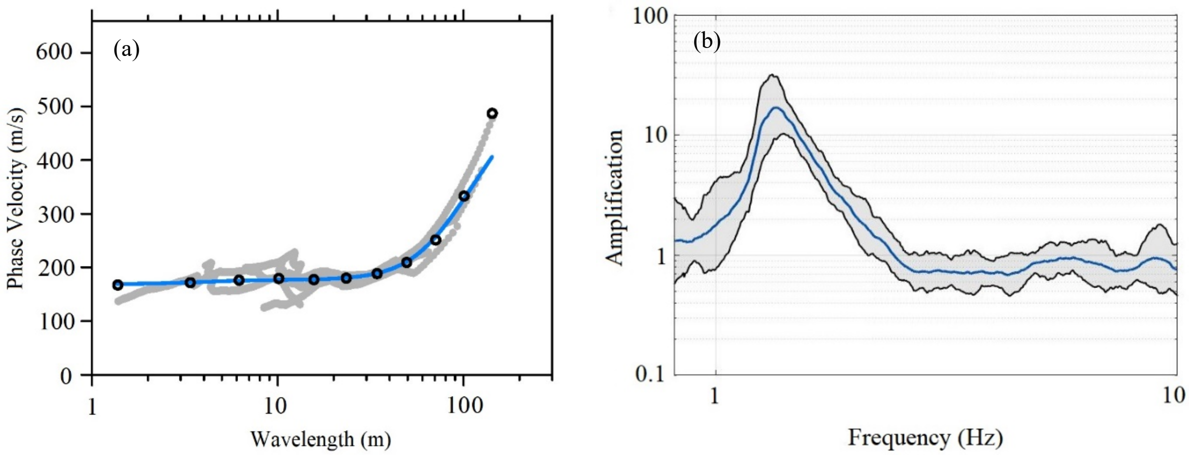

Figure 11: (a) Dispersion curve (Thompson et al. 2014), and (b) H/V spectral ratio measurements at Danehy park site

4.2. Inversion Results

An initial numerical model of the site geology was built assuming two layers of soil with depths of 25 and 5 meters, respectively. The assumption about the number and thickness of the layers are made based on previous analysis of the dispersion curves (Thompson et al. 2014) and is consistent with the generic soil profile for the area.

The proposed hierarchical Bayesian inversion method is then deployed using both sets of surface measurements. The shear wave velocity for each of layer was estimated along with their associated uncertainties. Posterior distributions of the shear wave velocities are plotted in Figure 12. As can be seen, uncertainty of the estimated shear wave velocity is larger for the bottom layer which can be due to fact that the surface wave measurements are generally less sensitive to the shear wave velocity of deeper soil layers.

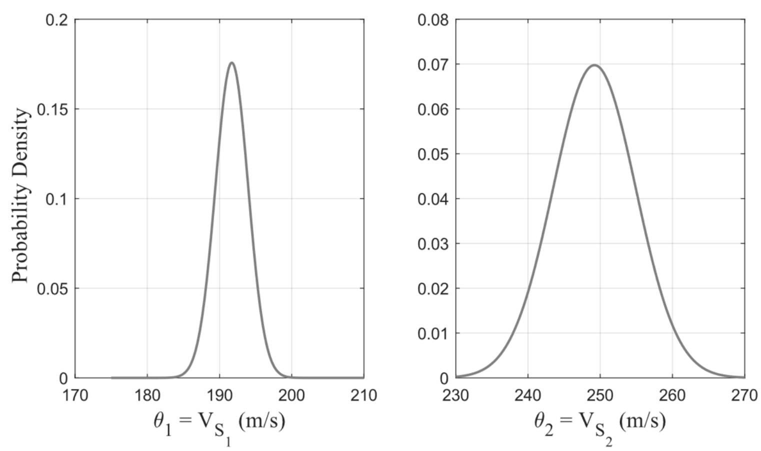

Figure 12: Posterior distributions of the shear wave velocity of the soil layers

Figure 13 shows the histograms for means of the estimated shear wave velocities. The black lines represent the kernel PDF which is normalized to match the highest bins of the histogram and black dots are the estimated MAP values of the distribution.

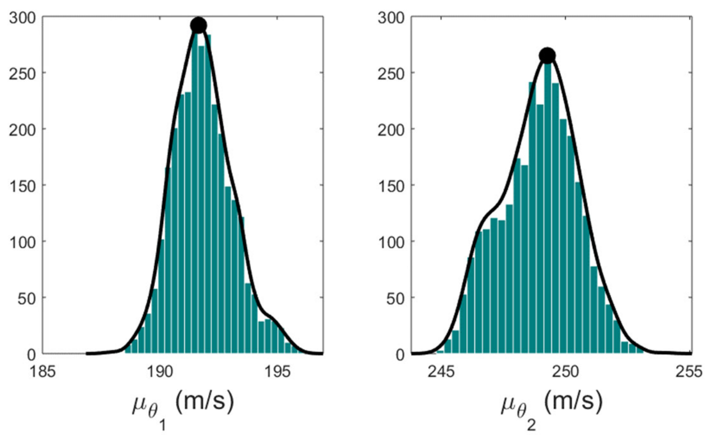

Figure 13: Histogram of posterior distribution for mean values of shear wave velocity

The histograms and normalized kernel PDFs of the posterior distribution for standard deviations of the estimated shear wave velocities are shown in Figure 14. A similar procedure is used for plotting these kernel distributions and for estimating the MAP values. These plots show that not only the values (as observed in Figure 12) but also the uncertainties of the estimated mean and standard deviations increase for the lower layer. This further underlines the importance of having a larger dataset comprised of complementary measurements to improve the estimation uncertainty of results.

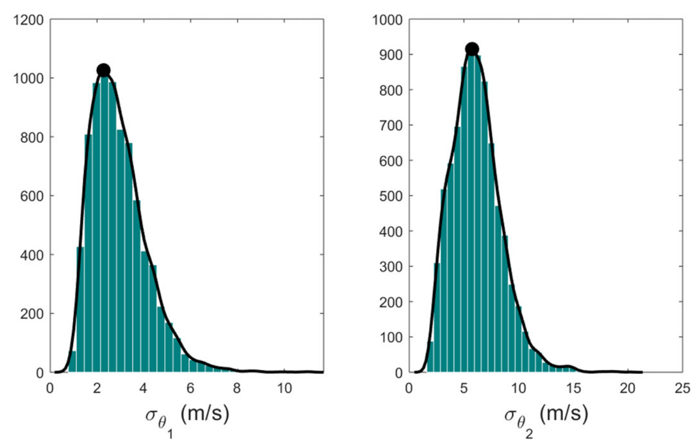

Figure 14: Histogram of posterior distribution for standard deviation of shear wave velocity

5. Summary and Conclusions

The hierarchical Bayesian inversion framework proposed in this project was studied for different numerical cases as well as an experimental case. It was observed that the proposed hierarchical inversion framework performs better at estimating the uncertainty associated with the shear wave velocity compared to the classical Bayesian methods. While the uncertainty estimates from the classical Bayesian inversion are highly sensitive to the amount of data and go to zero with increasing datasets, the uncertainties estimated using the hierarchical inversion converge with more data and provide realistic values even for a small number of dataset (e.g., 5 datasets in the numerical study here). Increasing the amount of data will increase the confidence in the updating parameters and predictions obtained from the updated models. Effect of including the fundamental frequencies from the H/V ratios to the phase velocities from the dispersion curve measurements are also studied. It was noticed that additional of data H/V ratios reduced the uncertainty associated with the estimated hyperparameters, in this case mean and standard deviation of the shear wave velocities. Given that collecting ambient vibration measurements to calculate H/V ratios is relatively cheap and easy, this improvement can be very promising. To further investigate the efficacy of the proposed method, it was employed for a site in Boston area. The hierarchical inversion method was used to estimate the distribution of shear wave velocities while quantifying the uncertainty of distribution parameters estimated. Further study using larger datasets are needed for different geological sites in order to better understand the effectiveness and practicality of adding more surface measurements and also including H/V data in the inversion process.

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