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Title: Ultrasonic investigations of the micro-mechanical origin of rate and state friction

PI: Allan Rubin
Department of Geosciences
Princeton University, Princeton, N.J. 08544
Phone: (609) 258-1506
Fax: (609) 258-1274
e-mail: arubin@princeton.edu

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Abstract

Traditional descriptions of state evolution in the rate-state friction framework, e.g. the Dieterich (Aging) and Ruina (Slip) laws, do not explain the whole range of laboratory friction experiments. *Nagata et al.* (2012) have recently suggested that adding a shear stress dependence to Aging law state evolution, derived using ultrasonic P-wave transmissivity across frictional interfaces in granite as a proxy for the state variable, could remedy these deficiencies. For 1-3 orders of magnitude velocity step datasets on both rock and gouge, each dataset fit well by the Slip but not the Aging law, the shear stress dependent Nagata state evolution produces fits identical to the Slip law as long as the stressing-rate parameter c is larger than a lower bound. This lower bound on c ranges from $\sim 10 - 70$ over 2- to 3-orders steps as opposed to the $c = 2$ reported by *Nagata et al.* (2012) for their generally smaller departures from steady state. On the other hand, the Nagata law with $c = 2$ fits the stress minima at the end of a sequence of holds better than both the Aging and Slip laws but fails to match the evolution of peak stresses upon reslide after long holds. These disagreements with laboratory data and the variations in c under different mechanical conditions leads us to question both the mathematical form of the Nagata stressing-rate dependence and the mapping between acoustic transmissivity and state, the latter assumed to be linear by Nagata et al. [2012]. We investigate these issues using a set of large velocity steps, long slide-hold-slides (SHS) and normal stress-steps on bare rock complemented by ultrasonic monitoring of the frictional interface. Unlike the experiments of *Nagata et al.* (2012), we study the micro-mechanical details of frictional sliding far from steady state by using P-wave transmitted phases. It is observed that the values of c for the best Nagata law fits to the stress data, which also lead to the best agreement between the Nagata law state variable and acoustic transmissivity, differ by almost an order of magnitude between large velocity steps and SHS tests during the same experimental run. The Nagata law, however, fails to match both the stress peaks and the evolution of transmissivity across reslides after long holds. The results indicate that while the evolution of ultrasonic transmissivity shows a robust shear stress dependence, the mapping of the ultrasonic data into variations of 'state' is less certain. Our study also reveals that the mathematical form of the stressing-rate dependence of state evolution suggested by *Nagata et al.* (2012) needs to be revised to explain the full range of mechanical and ultrasonic observations. These results helped us formulate strategies for future experiments and the development of theoretical tools to analyze the resultant data which formed the core of a currently funded NEHRP proposal.

Contents

1 Introduction **4**

2 Background **5**

3 Response to velocity steps **6**

3.1 Aging law - Slip Law transition with variation of c 7

3.2 Large velocity steps on gouge and bare granite: How large must c be? 9

4 How well can existing state evolution laws explain Slide Hold Slides? **11**

5 Ultrasonic monitoring of contact interfaces **13**

6 How is ultrasonic transmissivity related to state **15**

7 Future work **17**

8 Conclusions **17**

9 References **19**

1 Introduction

Our understanding of the role of aseismic transients in influencing mechanical conditions along major plate boundaries has changed considerably due to the discovery of episodic tremor and slip phenomena (*Peng and Gomberg*, 2010; *Schwartz and Rokosky*, 2007). Additionally, analysis of inter-plate mainshocks in the North Pacific suggest that foreshocks may be driven by extended-duration aseismic slip (*Bouchon et al.*, 2013), an observation which has parallels in laboratory nucleation experiments on meter-scale synthetic faults (*McLaskey and Kilgore*, 2013). Foreshocks have been observed to elevate the regional seismicity rate over a period of 1 to 20 days before major inter-plate mainshocks, and the size of foreshock zones of some M4.7–7.3 earthquakes along the San Andreas fault have also been observed to scale with the mainshock moment (*Dodge et al.*, 1996). These observations highlight the fact that realistic predictive modeling of these aseismic transients would be a significant breakthrough in both earthquake hazard estimation and mitigation. Any numerical model of the nucleation process needs to use the 'proper' description of fault friction to be relevant to realistic hazard assessment.

Rate and state friction (RSF) is our most widely used set of constitutive equations for describing fault friction. Robust laboratory observations have established the dependence of friction on slip rate and 'state' (some measure of the quality and/or quantity of true contact area). Unfortunately, we still lack a single friction constitutive law which can explain the full range of laboratory friction observations. This severely hampers realistic seismic hazard assessment because differences in the friction law can lead to markedly different styles of earthquake nucleation in numerical models, extending to even the extent to which earthquake nucleation is geodetically observable (*Ampuero and Rubin*, 2008; *Bhattacharya and Rubin*, 2014). These different nucleation styles arise from the differences in the response of the various friction laws to large perturbations from steady state sliding (*Ampuero and Rubin*, 2008; *Bhattacharya and Rubin*, 2014). But friction experiments often design and test friction laws with much smaller excursions from steady state. To test the friction laws in this more relevant portion of parameter space, we carried out a suite of velocity step and slide-hold-slide (SHS) experiments (on initially-bare rock and gouge samples) designed to push the sliding surface as far from steady state as possible.

These mechanical experiments were accompanied by ultrasonic monitoring of the sliding interface to probe the micro-mechanics of sliding contacts. The discipline of non-destructive evaluation has long used ultrasonic monitoring techniques to measure mechanical properties of interfaces in opaque materials (*Nagy*, 1992). *Nagata et al.* (2008, 2012) adopted ultrasonic monitoring to study the quasi-statically sliding interface of bare rocks and measured P-wave transmissivity across the slip surface as a proxy for real contact area. For decades, it had been assumed that the evolution of fault state requires the passage of either slip or time. Arguing that acoustic transmissivity (AT) across the slip surface is linearly related to fault 'state', *Nagata et al.* (2012) used their P-wave transmissivity data across smaller departures from steady state to suggest that state also evolves with shear stress. We used existing datasets comprising larger departures from steady-state sliding (one- and two-order-of-magnitude velocity steps and sequences of slide-hold-slides) to check if this new state evolution law was better at explaining laboratory friction data than the prevalent, widely used versions (*Bhattacharya et al.*, 2015, 2014). While the Nagata law produced the best fits of all the state evolution laws we considered, disagreements remain with some robust features of the data (*Bhattacharya et al.*, 2015, 2014). These disagreements motivate us to ask if the mathematical form of the stressing rate dependence needs revision. Our experiments are well suited to addressing this question as they are designed to constrain the transmissivity – 'state' relationship across a more diverse set of mechanical conditions than *Nagata et al.* (2012). The ultimate goal of these experiments are to motivate a sound theoretical footing for the existing,

largely empirical, friction prescriptions at quasi-static slip rates thus aiding the formulation of the micro-mechanically 'correct' constitutive equations for fault friction.

2 Background

RSF describes the frictional strength of a sliding interface as a function of two variables: The sliding rate, V, and the 'state variable', θ – traditionally thought of as a proxy for true contact area between the sliding surfaces (*Nakatani*, 2001). These variables are related by two, coupled, first order ODE's. The first of these equations describes the relationship of the frictional strength to the rate and state variables and is called the friction law –

$$\frac{\tau}{\sigma} = \mu_* + a \log \frac{V}{V_*} + b \log \frac{\theta}{\theta_*}, \tag{1}$$

where a is the 'direct effect' coefficient controlling the magnitude of velocity-dependence, and b is the 'evolution effect' coefficient controlling the magnitude of state-dependence. In general, a and b are constants of the order of 0.01 but can vary with temperature and moisture content (*Blanpied et al.*, 1998). μ_*, V_* and θ_* are the values of friction coefficient, slip rate and state at some reference steady-state. The second differential equation describes the evolution of θ and the two most widely used forms are

$$\text{Aging Law} : \dot{\theta} = 1 - \frac{V\theta}{D_c} \tag{2a}$$

$$\text{Slip Law} : \dot{\theta} = -\frac{V\theta}{D_c} \ln \frac{V\theta}{D_c} \tag{2b}$$

where D_c is some characteristic slip weakening length scale. Eq. (2a) is often referred to as the Aging law while Eq. (2b) is referred to as the Slip law (*Dieterich*, 1978; *Ruina*, 1983). State evolves only at non-zero slip rates for the Slip law while for the Aging law it can evolve both with slip (second term on the right) and time (first term).

It has become clear over the years that neither the Aging law nor the Slip law accurately describe all laboratory friction experiments (*Beeler et al.*, 1994; *Kato and Tullis*, 2001; *Bayart et al.*, 2006). For sufficiently stiff systems, the frictional response of bare rock or gouge to a step change in load point velocity shows that the slip weakening length scale is independent of the magnitude or sign of the step (see Figure 4) (*Ruina*, 1980, 1983; *Marone*, 1998; *Blanpied et al.*, 1998). Following such a velocity step, the Slip law predicts exponential slip-weakening behavior over a length scale that is independent of the size and sign of the velocity step, consistent with these observations (*Rice*, 1993; *Nakatani*, 2001). The Aging law, however, predicts linear slip weakening, with slope $b\sigma/D_c$, for large velocity increases. For this reason the evolution of the frictional strength to steady state occurs over length scales which increase with the size of the jump (*Ruina*, 1980; *Nakatani*, 2001). For large velocity step decreases, the Aging law response evolves to steady state over slip distances much smaller than D_c. Therefore, unlike the laboratory data in Figure 4, the Aging law predicts very different characteristic length scales for state evolution in response to large velocity step ups and downs.

Rubin (2008) argued that the Slip law is more appropriate for simulating earthquake nucleation, because it accurately describes existing laboratory behavior near to and far above steady state, where most of nucleation occurs. But the Aging law, due to its time dependent healing property, has been claimed to be better suited to describing fault healing during interseismic periods (*Beeler*

et al., 1994; *Marone et al.*, 1995; *Marone*, 1998). In fact, both laws are deficient. It is often said that the Slip law does a better job of matching velocity step tests, whereas the Aging law, with its time-dependent healing term, does a better job of modeling the healing that occurs during the hold portions of SHS tests. In section 2.4 we present new analytical results and a reinterpretation of the data of *Beeler et al.* (1994) that cast doubt on the assertion that frictional healing during long holds is more consistent with the Aging than Slip law. Ideally, one would hope for an evolution law that could simultaneously explain all the robust features of velocity step and SHS tests, but what mathematical form of the state evolution law will lead us there is still an open question (*Kato and Tullis*, 2001).

Recently, *Nagata et al.* (2012) carried out a series of 'shear stress step' experiments on initially bare rock and measured the variation in AT across the slipping interface. They assumed that AT varies linearly with real contact area and, using this 'direct' measure of state, estimated the direct effect coefficient a to be ~ 0.05, much larger than the usual estimate of $a \lesssim 0.01$. Both the Slip and Aging laws failed to fit their laboratory data with such a large value of a. Instead, to fit their experimental results, *Nagata et al.* (2012) modified the Aging law by including a dependence upon stressing rate:

$$\text{Nagata Law}: \dot{\theta} = 1 - \frac{V\theta}{D_c} - \frac{c}{b}\theta\frac{\dot{\tau}}{\sigma}. \tag{3}$$

Here, c is another parameter and all other symbols carry over their respective meanings (but not necessarily values) from equations (1) and (2a). For $c = 0$, this law reduces to Aging law state evolution. *Nagata et al.* (2012) found $c = 2.0$ to be an appropriate value for their experiments. The authors claimed with numerical simulations that the modification with the stressing rate term led to nearly symmetric response to velocity step tests (*Nagata et al.*, 2012). When viewed alongside the fact that equation (3) retains the healing term, this raises the possibility of the Nagata law being able to explain all the robust observations from rock friction experiments.

3 Response to velocity steps

Under the Slip and Aging laws, when a fault surface is subjected to a step change in sliding velocity, there is no instantaneous change in state. The stressing rate term in the Nagata law, however, does produce such an instantaneous change. The exact expression for the variation of the stress change with slip following an instantaneous step in sliding speed is

$$\Delta\tau(\delta) = b\sigma \ln\left[1 - \left\{1 - \left(\frac{V_f}{V_i}\right)^{1-\frac{c}{c+1}\frac{a}{b}}\right\}\exp\left(-\frac{\delta}{D_c(1+c)}\right)\right], \tag{4}$$

where δ is slip since the velocity step, V_f and V_i are the final and initial slip rates, respectively, and $\Delta\tau$ is the stress relative to the future steady state value at slip speed V_f (*Bhattacharya and Rubin*, 2014). This expression has several interesting properties. In particular, with increasing values of c, the Nagata law response to velocity step ups becomes increasingly like the Slip law over an increasing range of step sizes. It turns out that we can capture these transitions analytically by asymptotic analysis of Eq. (4).

3.1 Aging law - Slip Law transition with variation of c

Bhattacharya and Rubin (2014) showed that, for any given value of c in Eq. (4), there exists a critical value of V_f/V_i above which the Nagata law response will show linear slip weakening behaviour, similar to the Aging law. The size of this critical velocity jump increases with the value of c. In this linear slip-weakening limit

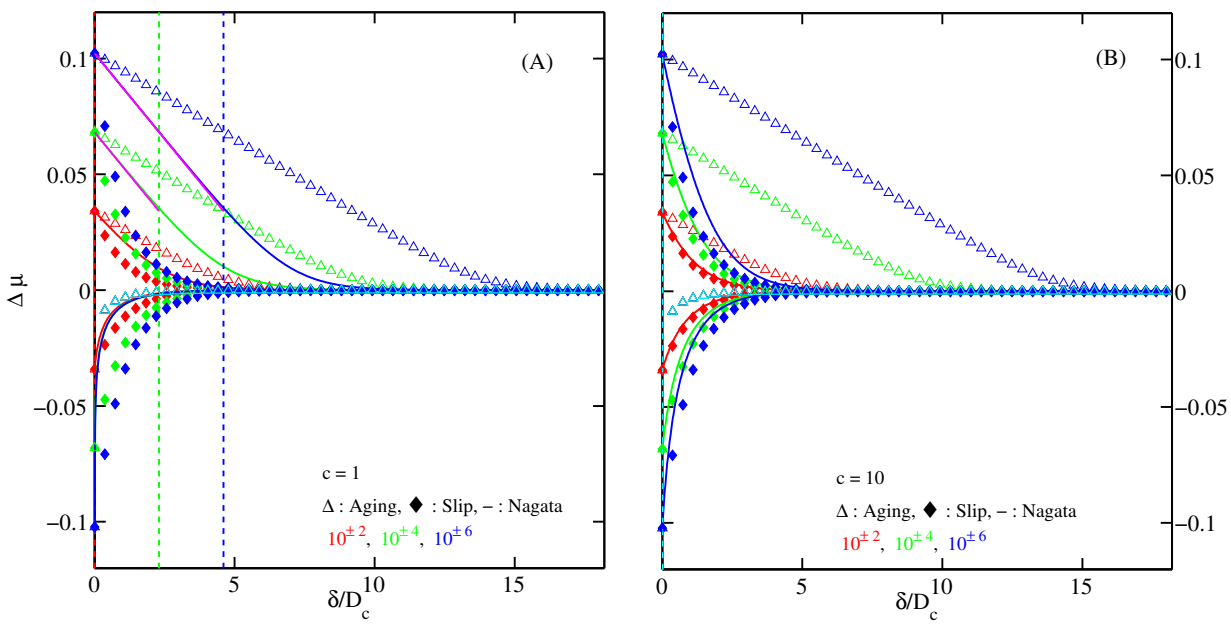

Figure 1: Analytical behaviour of the Aging law (open triangles), Slip law (solid diamonds) and the Nagata law (solid curves) for step ups/downs in slider velocity of magnitudes $10^{\pm 2-8}$ with different values of c: (A) $c = 1$, (B) $c = 10$. For the Aging and Slip laws $a = b = 0.007, D_c = 10\,\mu$m. The scaling relations in Eqs. (7a), (7b) and (7c) were used to obtain a, b and D_c for the Nagata Law. For non-zero c one observes Slip law-like behaviour for small velocity jumps but Aging law-like linear slip weakening response for larger jumps. The magenta lines superposed on the Nagata law curves in (A) are the slope estimates from the linear slip weakening approximation from Eq. (5). The magnitude of the velocity jump required to see the Nagata law transition between Aging and Slip behaviour increases as c increases. Colors correspond to the size of the jumps.

$$\Delta\tau \approx b\sigma\left(1 - \frac{c}{c+1}\frac{a}{b}\right)\ln\left(\frac{V_f}{V_i}\right) - \frac{b\sigma}{D_c(1+c)}\delta, \tag{5}$$

where the last term gives the linear slip weakening rate. However, in the limit $c \gg 1$, Eq. (4) reduces to

$$\Delta\tau(\delta) \approx b\sigma\left(1 - \frac{c}{c+1}\frac{a}{b}\right)\ln\left(\frac{V_f}{V_i}\right)\exp\left(-\frac{\delta}{D_c(1+c)}\right) \tag{6}$$

for any physically realizable size of the velocity step (V_f/V_i). That is, stress decays exponentially with slip over a characteristic distance independent of the sign or size of the jump. This is exactly the Slip law response given particular scaling relationships between the Nagata and Slip law parameters tabulated below. The stressing rate term in Eq. (3) adjusts how far above (or below) steady state the slip surface is pushed by a given velocity jump by controlling the amount of state evolution across the velocity step. Viewed in this way, the linear slip-weakening response occurs when c is small enough that the velocity step can push the slip surface far above steady state, while the exponential slip-weakening response occurs when c is large enough to prevent this. This

also means that any velocity step data from a sufficiently stiff testing machine that is well fit by the Slip law can be equally well fit by the Nagata law for any value of c greater than or equal to a lower bound. This lower bound is the smallest value of c for which Eq. (6) holds, given a particular V_f/V_i (*Bhattacharya and Rubin*, 2014). These transitions are shown in Fig. 1.

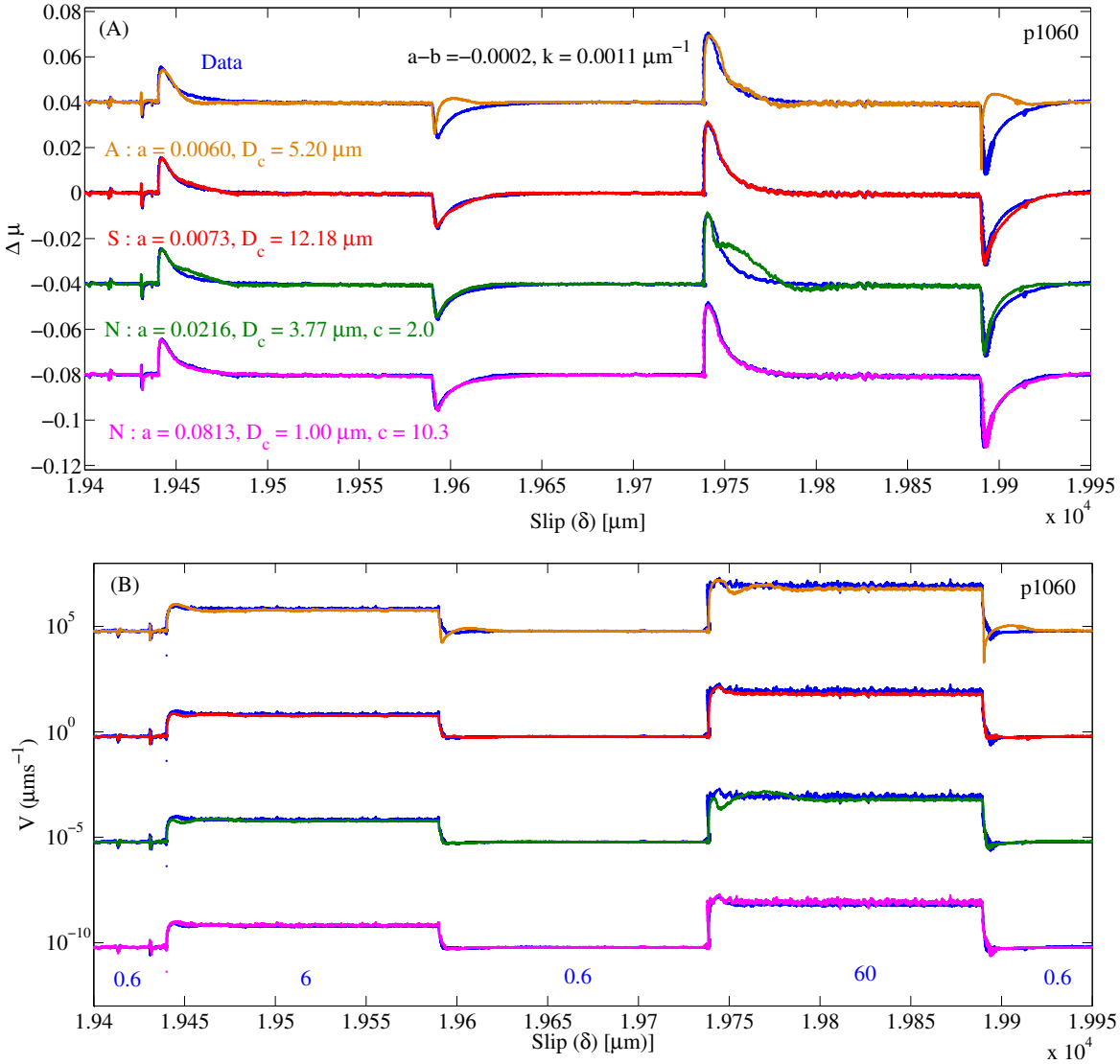

Figure 2: Results from a velocity step up/down experiment (dataset p1060) carried out by servo-controlling off a displacement transducer mounted directly on the sample, hugging the sliding surface. (A) The fit to $\Delta\mu$. (B) The corresponding slip rates (V) predicted by the forward model for the best fitting $\Delta\mu$ time series with the stiffness estimated as described in text. Blue: data; ochre: Aging law; red: Slip law; green: Nagata law with $c = 2$; purple: Nagata law with $c \approx 10$. The numbers in blue in (B) denote measured load point velocities in μms^{-1}. Plots scaled and shifted to improve visibility. The Slip law does a good job of fitting the data due to its characteristic exponential slip-weakening over a fixed length scale for step ups/downs of all magnitudes. The Nagata law does a slightly better job with $c \approx 10$.

To complete the discussion of the response of the Nagata law to velocity step tests, we tabulate three scaling relationships that describe the exact correspondence between the Slip and Nagata

law parameters when Eq. 6 holds:

$$D_c(1+c)|_{\text{Nagata}} \equiv D_c|_{\text{Slip}}, \tag{7a}$$

$$\frac{a}{1+c}|_{\text{Nagata}} \equiv a|_{\text{Slip}}, \tag{7b}$$

$$(a-b)|_{\text{Nagata}} \equiv (a-b)|_{\text{Slip}}. \tag{7c}$$

Eq. (7a) follows from the requirement to match the slip weakening distance. Eq. (7b) follows from the requirement of having the same peak stress value for the Nagata and Aging/Slip laws following a given velocity step up/down. Eq. (7c) follows from the requirement to have the same amount of steady state velocity weakening/strengthening independent of the state evolution law.

3.2 Large velocity steps on gouge and bare granite: How large must c be?

Given the importance to earthquake nucleation of the response of the fault surface to large and sudden velocity increases, we have revisited the one- and two-order-of-magnitude velocity step data of *Bayart et al.* (2006) (on synthetic gouge at 25 MPa normal stress) in light of our analytical results for the Nagata law discussed above. *Bayart et al.* (2006) showed that these data are modeled very well by the Slip law and very poorly by the Aging law. Because these experiments pushed the sliding surface much farther from steady state than the bare rock experiments of *Nagata et al.* (2012), our analytical results suggest that to fit these data well might require c to be larger than the values of $c \sim 2$ that *Nagata et al.* (2012) suggested.

To test this hypothesis we performed rigorous parameter inversions on this dataset using our in-house small world Markov Chain Monte Carlo global search with adaptive proposal distribution (*Rosenthal*, 2011; *Bai*, 2009a,b; *Guan et al.*, 2006). For most of our inversions we fix $a - b \approx -0.0002$ based on the behavior at steady state, and $k = 0.0011\,\mu m^{-1}$ based on the short-term stress response to the largest load point velocity increase (Fig. A1 in Appendix A of *Bhattacharya et al.* (2015)). The fits for the Aging, Slip and Nagata laws are shown in Fig. 2(A). Fig. 2(B) compares the slip velocity obtained by differentiating the fault slip calculated from the data ($V = d\delta/dt$) with the slip velocities predicted by the respective state evolution law fits shown in Fig. 2(A). Fig. 2(A) clearly shows that the Slip law fits the data well while the Aging law fares much worse. The Nagata law with $c = 2$ does a considerably worse job of fitting the dataset than the Slip law. In fact, Nagata law fits nearly identical to those produced by the Slip law were found only for $c \gtrapprox 10$. This is demonstrated by the posterior distributions of a, D_c and c constructed from a Nagata law Markov chain (Fig. 3(A), (B) and (C)). In particular, the posterior distribution of c shows the presence of a peaked region corresponding to $c \sim 10-100$ and a quasi-uniform tail which continues to orders of magnitude larger values of c. *Bhattacharya et al.* (2015) have established that the sharp truncation of the posterior near $c \sim 10$ sets this value as the requisite lower bound on c for the Nagata law to fit the data as well as the Slip law. The slow fall off the quasi-uniform tail of the posterior for large values of c is due to the fact that our chains were truncated at some pre-decided upper bound of c by imposing constraints through the prior distribution.

To determine if we infer a much larger value of c simply because we pushed the sample farther from steady state, or if there is in fact a real difference between gouge and bare rock, we carried out a series of experiments in which velocity step increases of up to two orders of magnitude and decreases up to three orders of magnitude were imposed on granite blocks. The Penn State biaxial double-direct shear apparatus was used along with ultrasonic monitoring of the slip interface; the ultrasonic data will be discussed later. Because bare rock is more difficult to stabilize than gouge, the experiments were conducted at 5 MPa normal stress. We used a downhill simplex method

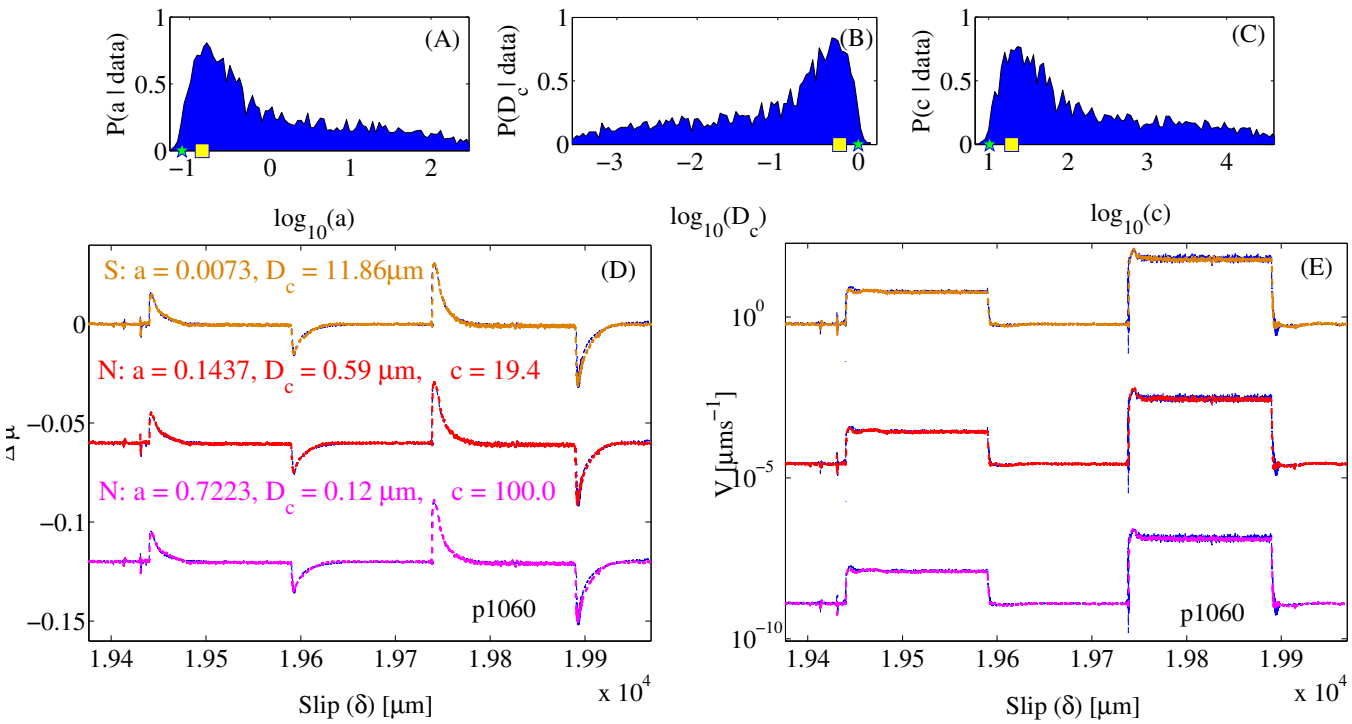

Figure 3: The marginal posteriors for the Nagata law Markov chain for the dataset in Figure 2 for (A) a, (B) D_c and (C) c. The first 2500 accepted samples were discarded to account for a burn-in period. The Markov sampling was truncated at $c = 10^5$. The posteriors are dominated by a locally peaked region and a quasi-uniform tail showing that there exist infinitely many solutions at this level of data error. The green star denotes the initial point and the yellow square the least RMSE value. (D) The fit to $\Delta\mu$ values and (E) The corresponding slip rates (V) predicted by the forward models which fit the stress data. Blue: data; ochre: the best fitting Slip law; red: Nagata law with $c \approx 19.4$ which leads to the least RMSE in the Markov chain; pink: Nagata law with $c = 100$ fits the data almost exactly like the Slip law. Exact values of a and D_c for this value of c were searched by redoing a downhill simplex with the values of c and $a - b$ fixed.

to locate the best fitting solutions with the Aging, Slip and Nagata laws with stiffness = 0.0075 μm^{-1} (obtained from the short-term stress response to the reslide following a 1000 s hold imposed later during the same experimental run) and $a - b \sim -0.0003$ (obtained from steady state sliding following the velocity steps). This dataset is again much better explained by the Slip law than by the Aging law, both when the misfit is weighted to impose equal penalties for all the steps, and alternatively with almost no penalty for mis-predicting the velocity increases. The latter fit is shown in Fig. 4. Note that the Aging law fit requires a very large D_c because Aging law evolution following large velocity decreases occurs over slip distances much smaller than D_c. This required D_c is far too large to fit the step increases (note the linear slip weakening with $\sim$ identical slopes in Figs. 4A, C, D, F, and H). The smallest value of c with which the Nagata law could produce a fit similar to the Slip law was $\sim$15.0, larger than the value of 10.0 in Fig. 2 because the departure from steady state is larger in Fig. 4. As with the Aging law, choosing $c = 2$ produces much too slow a stress decay following the step velocity increases to match the data. Therefore the large value of c required to fit the data in Fig. 2 does not reflect simply a difference between rock and gouge, and our bare-rock value of c for large departures from steady state seems inconsistent with that of *Nagata et al.* (2012).

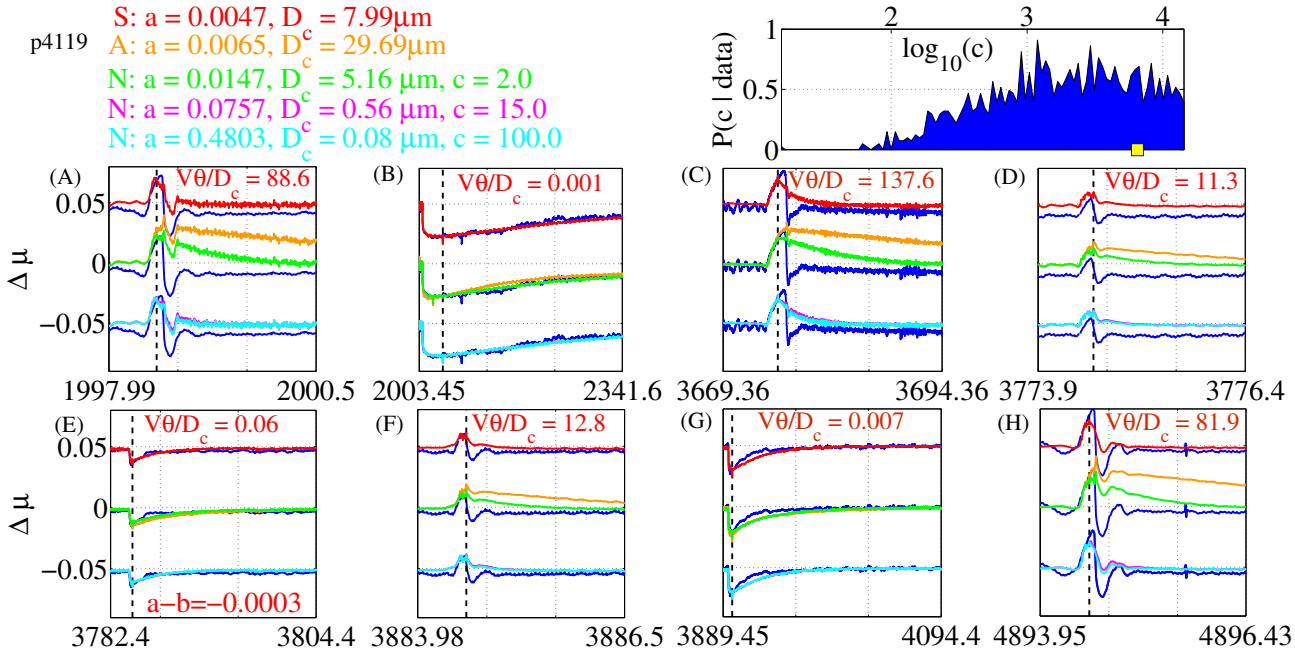

Figure 4: Results from load-point velocity step tests of (A) +2, (B) -3, (C) +2, (D) +1, (E) -1, (F) +1, (G) -2, (H) +2 orders of magnitude on bare rock showing the fits to $\Delta\mu$ using data from only the step downs: Blue-Data; Red-Slip law; Ochre-Aging law; Cyan-Nagata law with $c = 15.0$. Plots shifted to improve visibility. The Slip law does a good job of fitting the data due to its characteristic exponential slip-weakening over a fixed length scale for step ups/downs of all magnitudes. The Nagata law does an equally good job with $c = 15.0$. The values of $V\theta/D_c$ are its maximum (for a step up) or minimum (for a step down) following the particular step and are color coded according to the state evolution law (Slip or Nagata) used to infer them. In all cases stiffness (k), normalized by normal stress, is fixed at $0.0075\mu\text{m}^{-1}$. Top right: Posterior for c given the data, shows quasi-uniform tail representing all Nagata law fits to the data identical to the best fitting Slip law.

4 How well can existing state evolution laws explain Slide Hold Slides?

Over the last two decades, it has been widely held that the Aging law is better at explaining SHS tests on rock than the Slip law. The major laboratory evidence for this statement comes from a set of rotary shear experiments on initially bare granite by *Beeler et al.* (1994), wherein a sequence of SHS tests with holds from 10 to 10^5 s were repeated for two different stiffnesses. The peak stress after the reslide was found to increase logarithmically with hold time across the sequence of holds with a slope independent of stiffness, a feature shared by the Aging law but not by the Slip law. It is worth emphasizing that the main purpose of these experiments was to determine the amount of frictional healing (increase in state) during the holds. The peak stress upon reslide was used only as a proxy for this healing, there being no way to estimate 'state' at the end of the hold more directly. However, using peak stress for this purpose requires assuming that there is negligible state evolution between the start of the reslide and the attainment of peak stress, and there are two processes that could render this assumption invalid: (1) If the testing machine is not infinitely stiff, both the Aging and Slip laws predict that there will be some state evolution between the start of the reslide and peak stress. This effect is rather small and in any case is easily accounted for using numerical models. (2) Even if the Aging or Slip law correctly characterizes state evolution during the hold, state may evolve between the start of the reslide and peak stress in a way that is not captured by these laws. This effect cannot be accounted for by numerical models without knowing the proper evolution law. That some such mechanism may be operating was suggested by the lucite experiments of *Dieterich and Kilgore* (1994), who

showed (their Figure 7) that there was significantly more loss of contact area (transmitted light) between the end of the hold and the attainment of peak stress than would be predicted by state evolution under the Aging or Slip laws. The shear stress dependence of the Nagata law is one such mechanism by which this could occur.

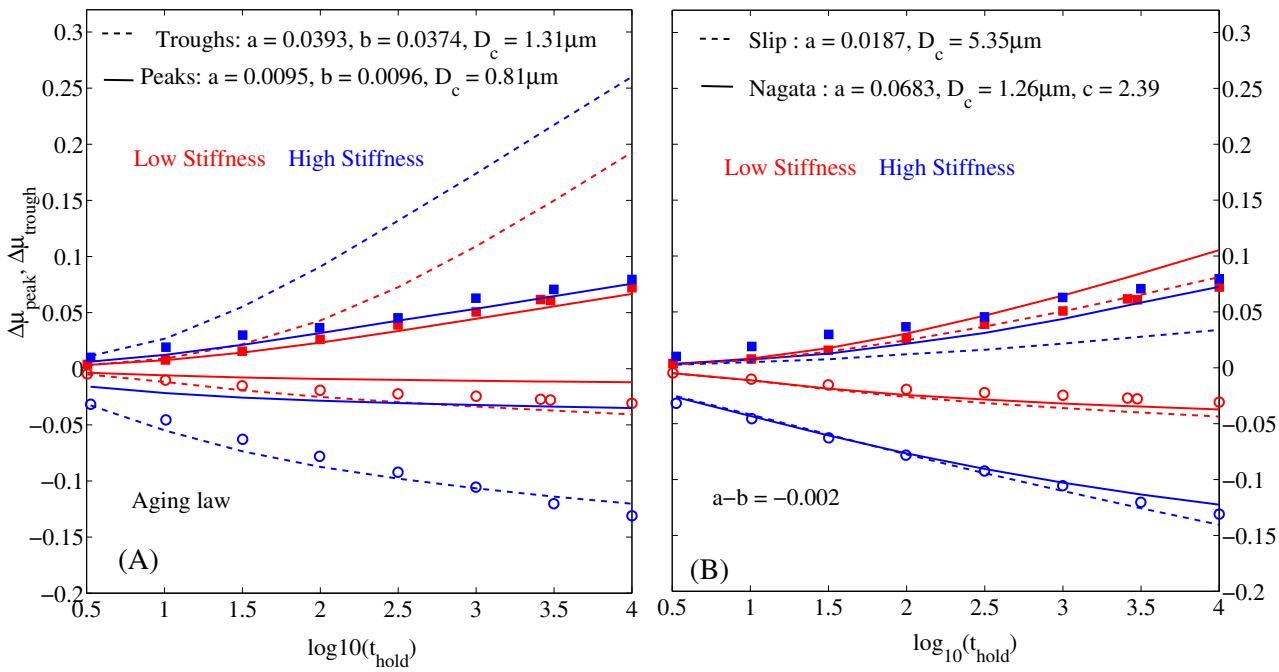

Figure 5: Results of inversions constrained to fit both the stress minima ($\Delta\mu_{\text{trough}}$, circles) at the end of the hold and the peak stress ($\Delta\mu_{\text{peak}}$, solid squares) after reload with the Aging, Slip and Nagata laws. Red colors denote low stiffness while blue colors denote high stiffness data and models. (A) Aging law fits with no constraint on $a-b$. Solid lines are predictions of both peaks and troughs based on fits to only the peak stresses, while the dashed lines are predictions based on fits to only the stress minima. (B) Slip (dashed) and Nagata law (solid) fits to both peaks and troughs simultaneously with the constraint $a-b=-0.002$ from previous velocity steps.

To dispense with the assumption of no state evolution prior to peak stress, we re-examined the data of *Beeler et al.* (1994) but focussed on the stress at the end of the hold. In work conducted under our current NEHRP grant we showed analytically that under the Aging law, stress at long hold times asymptotically approaches a constant value for velocity weakening materials and a nearly constant value (actually a decrease as log[log hold time]) for velocity neutral materials (*Bhattacharya et al.*, 2014) (equations 8 and 9 of *Ranjith and Rice* (1999) are correct but the statement in their equation 32 that these necessarily imply $\tau \to -\infty$ as $\log(V) \to -\infty$ is not). Only velocity strengthening Aging laws lead to ever decreasing stress minima with increasing hold durations, but even then the decrease is stiffness independent. In contrast, under a velocity weakening Slip law, stress continually decreases with log time for even the longest holds. This decrease in stress is stiffness dependent. One dominant characteristic of the *Beeler et al.* (1994) data is the continual decrease of stress with log hold time, most strongly for the high-stiffness dataset where (for zero load-point velocity) a given sliding speed gives rise to a larger stress reduction. From our analytical results, this behavior seems more consistent with the Slip law than with the Aging law, more so because velocity steps in the same experiment show this surface to be velocity-weakening.

To explore this further, we used our MCMC algorithm with the Aging, Slip and Nagata laws

to analyze the sequence of stress peaks and/or minima under a joint inversion scheme for both stiffnesses. When forced to fit only the stress minima, the only possible Aging law solution is velocity strengthening and it drastically overestimates the stress peaks upon reload (Fig. 5A, dashed lines). Note also that although the fit to the average stress minimum for the high-stiffness experiment is not terrible, it achieves this by underestimating the stress for short hold times and overestimating it for long hold times, a flattening trend not seen in the data but consistent with the analytical result of an approach to constant stress. When fitting the peaks only, the best fitting Aging law is slightly velocity weakening and leads to stress minima that are far too small (in absolute value) and that decrease far too slowly to match the data (Fig. 5A, solid lines), again consistent with our analytical results. Furthermore, both these fits violate the knowledge, from prior velocity steps in the same experimental run, that the granite surface is velocity weakening ($a-b=-0.0002$). Therefore, contrary to widely held belief, the data of *Beeler et al.* (1994) do not support time-dependent healing during holds as embodied by the Aging law. This is consistent with the failure of the Aging law to properly model large velocity step decreases, in that (as with load point holds) the sliding surface quickly enters a regime where the Aging law becomes $\dot{\theta} = 1 - V\theta/D_c \approx 1$.

In contrast, even when constrained to use the known value of $a-b$, both the Nagata and Slip laws do a much better job of simultaneously fitting the stress peaks and the continually-decreasing stress minima at the ends of the holds (Fig. 5B). Although not shown in Fig. 5, we note that we also fit the data using the Kato-Tullis correction to the Slip law that adds a time-dependent healing term at slip speeds below a critical value V_c: $\dot{\theta} = e^{(V/V_c)} - V\theta/D_c \log(V\theta/D_c)$. Given the extra free parameter V_c, the inversion chooses to make V_c lower than the lowest modeled slip speeds during the holds, meaning that these data are fit best when the Kato-Tullis correction plays no role. The best-fitting Nagata law does better than the Slip law and fits the stress minima better than even the Aging law fit to the stress minima only. Moreover, the best-fitting value of c is ~ 2, consistent with the bare-rock value obtained by *Nagata et al.* (2012). But it predicts that the evolution of peak stress with hold time is stiffness dependent, and that the higher stiffness machine produces the lower peak stresses. Both these features are in disagreement with the data and indicate that the Nagata law, although it does the best job of the laws tested, does not model the reslides well. Our proposed work aims to get to the bottom of this discrepancy by using AT to understand what happens during rapid velocity increases, which (we reiterate) exerts the dominant control on earthquake nucleation in numerical simulations.

5 Ultrasonic monitoring of contact interfaces

A major part of our proposed work was focused on complementing the mechanical (stress and displacement) data (e.g. those shown in Fig. 4) with ultrasonic monitoring of the contact interface. We have conducted runs with the ultrasonic apparatus and have gained valuable insight, both on experimental logistics and numerical expertise required to analyze the information obtained. The ultrasonic monitoring setup is in regular use in the Penn State laboratory (*Knuth et al.*, 2013; *Kaproth and Marone*, 2013, 2014). In our experiments we used bursts of continuous ultrasonic measurements produced via two 0.5 inch shear-wave piezoelectric transducers (500 kHz, lead-zirconate-titanate). The source transducer was excited with a 900 V pulse. The transmitted P waves were received by an identical PZT transducer in the block opposite to the source. These waves were recorded with a GaGe CS8382 multichannel digitizer at 25 MHz (14 bit). We shot 100 waveforms per second to maintain the same temporal resolution as the mechanical data across departures from steady state sliding. To avoid overflowing of the recorder buffer, we mostly

recorded in bursts of ~ 1000 waveforms clustered around perturbations to steady state sliding and synced the mechanical data and the ultrasonic transducer using in-house codes at Penn State. To maintain the best resolution in time, we have not stacked waveforms. Figs. 6 and 7 show results from one of the experiment on granite. The variations in the transmitted P-wave amplitude corresponding to changes in the mechanical state of the sliding interface are evident.

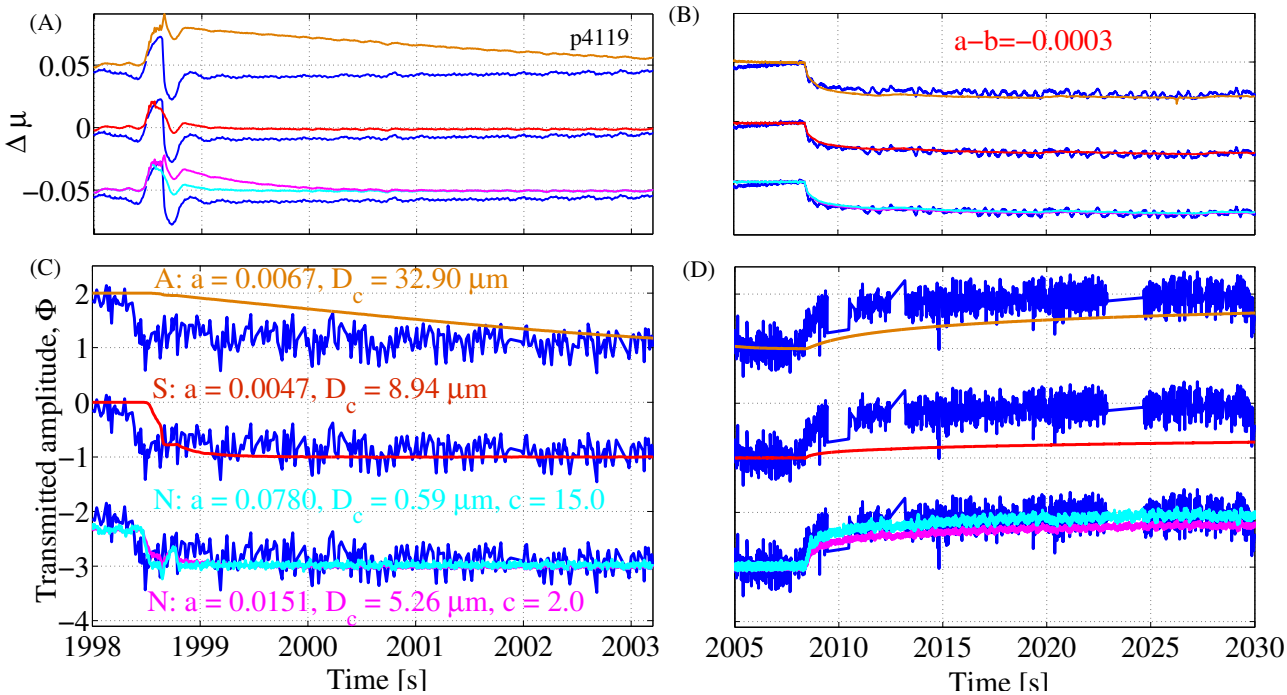

Figure 6: Results of ultrasonic monitoring of the sequence of the ~ 2 order step-up and ~ 3 order step-down from Fig. 4. (A) and (C) Fits to the friction data, with blue-data; ochre-Aging; red-Slip; pink-Nagata with $c = 2.0$; cyan-Nagata with $c = 15.0$. (C) and (D) Corresponding normalized P-wave transmitted amplitude in blue. Also shown are the estimates of $\Phi = \Delta\mu - a\log(V/V^*)$ obtained from the fits to the friction data. This figure fits only data from 1998 to 2030 seconds, a modest fraction of the data used in Fig. 4. That the model parameters are nearly the same points to the robustness of these fits. The normalized Φ values corresponding to the different evolution laws are color coded analogous to the friction data. Φ is widely interpreted as being proportional to contact area (*Nakatani*, 2001; *Nagata et al.*, 2012).

All our experiments show a clear and consistent signal in the transmitted acoustic amplitude time series. Generally, acoustic transmissivity (AT) fluctuates about $8-10\%$ in rock and $10-15\%$ in gouge in response to our velocity steps, holds and reslides. These fluctuations are observable even without stacking and are consistent across the whole range of mechanical conditions. One key assumption behind the conclusions of *Nagata et al.* (2012) is the linearity of the relationship between AT and true contact area. Following *Nakatani* (2001), they further relate the state variable $\Phi \equiv \Delta\mu - a\log(V/V^*)$ to contact area and, therefore, claim that Φ is linearly related to AT (we have normalized their definition of Φ with normal stress σ). To test this hypothesis, we used our numerical fits to the mechanical data to estimate Φ. To plot the data, we scaled the full range of both Φ and AT observations in Figs. 6C-D and 7E-H to the interval [0,1], with the data across all panels of each figure scaled jointly. The fits to the mechanical data show that the Nagata and Slip laws do an equivalent job of fitting the velocity steps (Fig. 6 with Nagata $c = 15$) while the Nagata law does the best job of fitting the SHS (Fig. 7 with Nagata $c = 2$). The Aging law does a considerably worse job than both its counterparts.

There are three important observations that come out of the Nagata law fits – 1) Both visually and in terms of the root-mean-square-error, the best fit to the velocity steps in Figs. 4 and 6 is provided by $c \gtrsim 15.0$, while for the SHS in Fig. 7, $c \sim 2.0$ is the best fitting value, similar to the value we obtained for the bare rock SHS data from *Beeler et al.* (1994). 2) Comparing the Nagata law estimates of Φ for $c = 15$ and $c = 2$, the value providing the best fit to the mechanical data also does the best job of tracking the fluctuations in AT for the mechanical regimes accessed in these experiments. 3) It is really striking that although the Slip and the Nagata laws can be tuned to produce nearly identical fits to the mechanical data, the Nagata law estimate of Φ does a much better job of matching the evolution of AT than its Slip law counterpart. In particular, our data show substantial evolution in AT immediately after the application of either a large velocity step or a reslide after a long hold, well before a peak stress is attained (see especially Fig. 7H). This feature is shared only by the Nagata law estimate of Φ because of the stressing rate term in the state evolution equation.

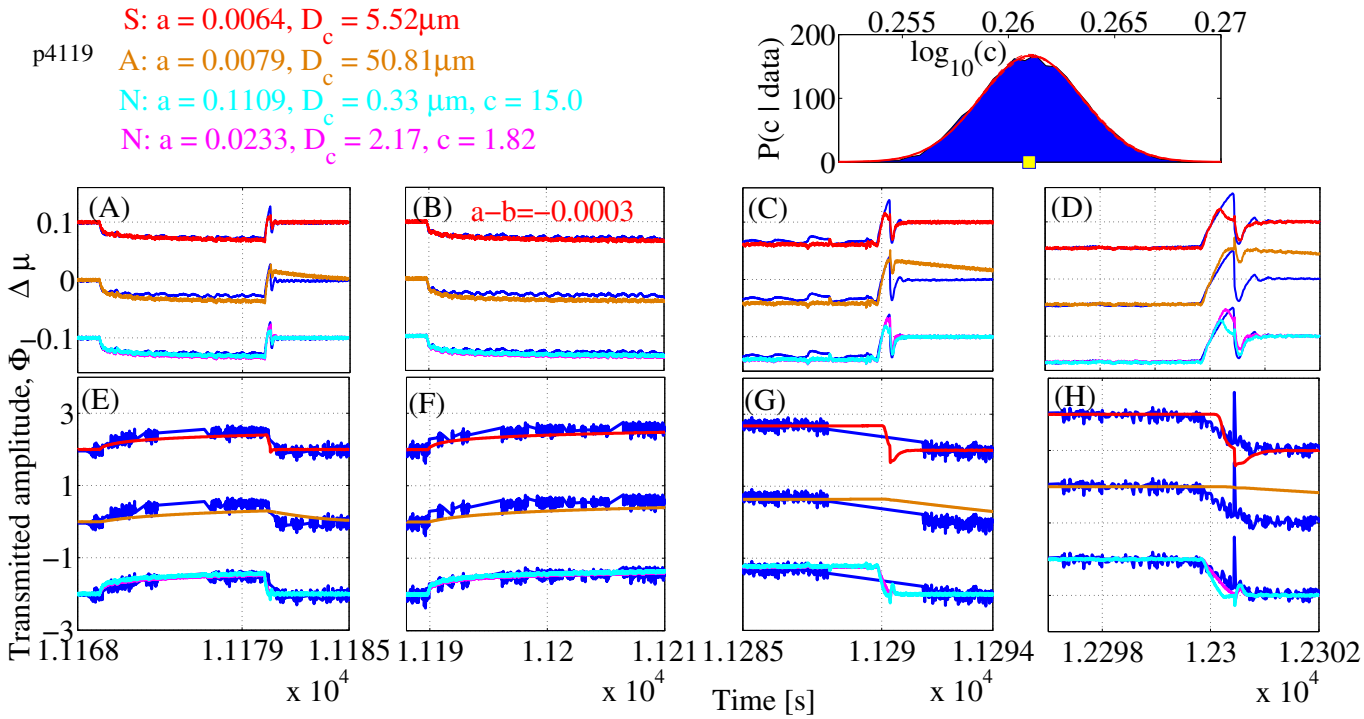

Figure 7: Ultrasonic monitoring of a sequence of SHS tests following the sequence of steps in Fig. 4. (A), (B), (C) and (D) show fits to the mechanical data, with blue-data; red-Slip law; ochre-Aging law; pink-Nagata with $c = 2.0$; cyan-Nagata with $c = 15.0$. $a - b = -0.003$ was fixed for these fits from the fit to the steps in Fig. 4. (E), (F), (G) and (H) show the scaled and shifted versions of S-wave transmitted amplitude in blue and Φ in the colors corresponding to the friction laws used to obtain the estimate. (G) is not useful across the reslide because of a mis-timed gap in ultrasonic data collection but it gives an indication of the full range of variation of AT. Top right: Posterior for c given the SHS data shows that the posterior is a tightly constrained Gaussian with $c = 1.82$ as mean. The solid red curve shows the maximum likelihood Gaussian fit to the posterior.

6 How is ultrasonic transmissivity related to state

The fact that one needs to vary c over almost an order of magnitude to produce the best fits to velocity steps and SHS in the same experiment indicates that, for state to have a direct stress dependence, the mathematical form of the shear stress dependence of state might need to be

revised. Moreover, our experiments also show that the evolution of the best-fitting Nagata law Φ is more rapid than that of AT following a reslide after a long hold (Fig. 7(H)). When the above points are viewed in conjunction with the fact that the Nagata law also fails to match the mechanical features of the reslide data in Fig. 5 it becomes apparent that the mathematical form of the Nagata law is inaccurate. To formulate necessary revisions, one needs to re-examine the two central assumptions that motivated the Nagata law state evolution – 1) Φ is linearly related to ultrasonic transmissivity; 2) AT shows a constant linear dependence on stressing rate over all magnitudes of velocity perturbations to steady state sliding. More experiments need to be performed to probe the veracity of both these assumptions.

The assumption that AT and real contact area are linearly coupled to each other throughout the whole range of laboratory friction experiments seems especially suspect in light of recent experiments on transparent plastic (lucite) by *Nagata et al.* (2014). In these experiments real contact area was optically monitored and AT was simultaneously recorded across velocity steps, SHS tests and normal stress steps on lucite. The experiments show that, during ~ 1 order-of-magnitude steps and holds, contact normal stiffness (numerically estimated from AT) faithfully tracked optically observed contact area (*Nagata et al.*, 2014). However, across reslides after long holds and normal stress steps, contact stiffness does not track optically observed contact area. For example, during normal stress steps, contact stiffness evolves faster across a stress increase than real contact area. What is even more remarkable is that, across the same normal stress step increase, shear stress evolves much slower than both contact stiffness and optically observed contact area. During reslides after long holds, contact stiffness first evolves faster than contact area, only for contact area to later evolve faster than contact stiffness (from prior to reaching peak stress and until reaching the subsequent steady state) (Nick Beeler, pers. comm.). These observations indicate that not only the relationship between AT (contact stiffness) and real contact area is most likely non-linear, but the time (and slip) scales over which both these quantities evolve might differ significantly from the corresponding properties of shear stress.

While one can question the applicability of lucite experiments to rock, it is nevertheless important to properly map ultrasonic transmissivity into contact area to be able to use this information in constructing a micro-mechanical understanding of friction. But given that the state evolution law only requires an equation for the evolution of $\Phi \equiv \Delta\mu - a\log(V/V^*)$, and that the relationship between Φ and real contact area is only conceptual, we will focus on the following question, more tractable for opaque materials – Can a formulation of Φ that is completely linearly related to our ultrasonic observations explain all the robust features of the mechanical friction data? Our experiments on granite show that Φ and AT are not linearly coupled across the reslide following the 1000 s hold, with Φ for the best-fitting c (~ 2) evolving more rapidly than AT from the onset of the stress increase till the subsequent steady state (Fig. 7(H)). The fact that optically observed contact area is also observed to vary more rapidly than AT before the attainment of peak stress after a reslide, makes this observation intriguing. This could also suggest that the tracking of AT by Φ during the experiments of *Nagata et al.* (2012, 2014) is the result of a linear approximation to an underlying non-linear dependence, valid only for small excursions from steady state, as they have suggested (e.g., Fig. 13 of *Nagata et al.* (2012)). One of our goals for future experiments is to verify whether our larger departures from steady state provide observational support for or against the linear relationship between transmissivity and stressing rate embodied by Eq. 3. By studying the large velocity steps and reslides, one would ultimately want to characterize the relationships between Φ, transmissivity and stressing rate both near to and far from steady state.

The question of whether the 'correct' evolution equation for Φ more closely tracks ultrasonic transmissivities or real contact area again leads to the issue of the relative importance of contact

'quality' vs. 'quantity' in determining the frictional strength of an interface. The importance of the lucite observations described by Beeler et al. is that they highlight the decoupling of ultrasonic transmissivity from real contact area. If indeed frictional strength is completely determined by a description of Φ which faithfully tracks transmissivity, distinguishing between the role of 'quality' and 'quantity' of contacts becomes equivalent to estimating the extent to which these ultrasonic parameters are functions of contact 'quality'. In fact, if the state variable is dependent on contact quality (related to bond strength), as suggested by *Li et al.* (2011), rather than quantity alone, simultaneous measurements of P- and S-transmissivities should allow us to probe such effects. This is one of our goals for future experiments.

7 Future work

We will continue our experimental philosophy of complementing our mechanical data with ultrasonic monitoring. Because we anticipate that the data will be simpler to interpret in terms of changes on the sliding surface, we expect to concentrate on bare rock. At a later stage, we will resume our experiments with gouge.

There is evidence that frictional processes on a discrete slip surface dominate bulk shear deformation in gouge after long run-ins, e.g. rapid changes in acoustic properties during the reslides after long holds are dominated by changes on (or near) the sliding surface (*Kaproth and Marone*, 2013). However, the processes involved in frictional sliding could be very different from bare rock if the frictional response of gouge to large stress changes is influenced by dilatancy or fabric formation off the main slip surface, which will also influence ultrasonic amplitude (*Kaproth and Marone*, 2014). Therefore the ultrasonic data might reveal ways in which the micro-mechanics of RSF in gouge is different from that in bare rock.

Our aim is to use both P- and S-wave amplitudes to study the micro-mechanics of friction. Both amplitudes carry information about the interface stiffness, but in orthogonal directions (with displacements parallel and perpendicular to the sliding surface). Simultaneous usage therefore yields more information concerning the state of the interface (*Baik and Thompson*, 1984; *Pyrak-Nolte et al.*, 1990; *Nagy*, 1992). In addition to a different elastodynamic response of the interface to P and S waves, differences may arise in the amount of inelastic deformation (dissipation) during wave transmission (e.g., sliding at the contact points is geometrically permissible; interpenetration of those contacts is not). For example, in their experiments on stationary (dry) surfaces, *Pyrak-Nolte et al.* (1990) found that their S-wave transmissivity data were fit best by allowing for a component of viscous deformation at the interface; such was not necessary to fit their P-wave data. Such micro-mechanical details of asperity interactions might hold the crucial clues towards designing the 'correct' friction constitutive equations.

8 Conclusions

We performed and analyzed a suite of experiments designed to verify the extent to which laboratory friction data agreed with the recently proposed shear stress dependent Nagata law state evolution. We have shown that for 1-3 order velocity steps on both gouge and rock, the Slip law fits the friction data consistently better than the Aging law. However, for all these datasets, the value of the shear stress dependence parameter c can always be tuned to be larger than a lower bound (generally of the order of 10 or larger) so that the Nagata law reproduces the best fitting Slip law. This lower bound is nearly an order of magnitude larger than the value $c \sim 2$ reported by *Nagata et al.*

(2012) for their experiments which comprised much smaller excursions from steady state. But, for slide-hold-slide tests performed during the same experimental runs, the appropriate value of c is found to be around 2 which is not only much smaller than what is needed to fit the large velocity steps but also similar to the value reported by *Nagata et al.* (2012). Such orders of magnitude variations in c observed between the fits to the two different types of experiments raises questions about the physical meaning of c and whether the linear shear stress dependence of state formulated in the Nagata law is indeed the correct description.

To further explore this issue, our recent experiments also involved acoustic monitoring of the sliding interface with ultrasonic P-waves similar to the original experiments of *Nagata et al.* (2012). We observed clear variations in transmitted acoustic amplitudes in response to imposed variations in sliding velocity. The values of c which lead to the best fits to the stress data also best explains the acoustic data. But the Nagata definition of state does not explain all the features of the acoustic data, especially across reslides after long holds. These disagreements are consistent with the independent acoustic and optical experiments of *Nagata et al.* (2014) and makes the linear mapping between state and acoustic transmissivity suspect. Further experimental work is needed to resolve these problems e.g. complementing the P-wave data with simultaneous S-wave observations might allow us to better understand the micro-mechanics at asperity contacts.

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