FINAL REPORT

EVALUATION OF GROUND MOTION SELECTION AND SCALING TECHNIQUES FOR LONG-PERIOD STRUCTURES

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1 Introduction

The results from this project fall into two main categories. They are briefly summarized below, and reported on in more detail in the following sections.

1.1 Testing of the proposed "Conditional Mean Spectrum" method

The PI has used this project to partially support his time working with the Pacific Earthquake Engineering Research (PEER) Center's Ground Motion Selection and Modification Program to evaluate a variety of motion selection and scaling methods. The PI will be a primary author on the comprehensive technical report that the Program will complete in April of this year. The PI's proposed "conditional mean spectrum" (CMS) ground motion selection and modification (GMSM) method is emerging as the preferred method from a comparison of dozens of proposed record selection schemes. As part of this project, a summary paper was published at the 8th Pacific Conference on Earthquake Engineering held last December in Singapore. It is reproduced as Section 2 of this report.

1.2 Extension of PSHA deaggregation calculations

An important step in future practical implication of the "conditional mean spectrum" (CMS) ground motion selection and modification (GMSM) method is to adapt it to be compatible with the probabilistic seismic hazard analysis (PSHA) tools provided by the USGS. A major research challenge in this task is to modify traditional PSHA deaggregation calculations to also provide conditional probabilities related to branches of the PSHA logic tree (e.g., "what is the probability that a Peak Ground Acceleration of 1g at site X was predicted by the Abrahamson & Silva ground motion model, as opposed to other ground motion models?"). These probabilities are needed for accurately executing the conditional mean spectra calculations. The details of these new calculations are provided in Section 3 of this report.

2 Measuring Bias in Structural Response Caused by Ground Motion Scaling

This section is a reproduction of a paper with the same name, presented at the 2007 8th Pacific Conference on Earthquake Engineering (http://www.ntu.edu.sg/cee/8PCEE/), and authored by the PI.

2.1.1 Abstract

The selection and scaling of earthquake ground motions is an important step in defining the seismic loads that will be applied to a structure during structural analysis, and serves as the interface between seismology and engineering. Recent research suggests that potential problems caused by ground motion scaling are due primarily to discrepancies in the shape of elastic response spectra between the ground motion to be scaled and the 'target' ground motion desired. These discrepancies may result in the scaled ground motions causing different levels of structural response than the response that would be caused by (unscaled) ground motions naturally at the intensity level of interest. A method for detecting such scaling bias is proposed, based on selecting a suite of ground motion records that have been scaled to all have the same intensity level (where here intensity is measured by spectral acceleration at the structure's first-mode period). The structural responses associated with the records are plotted versus the records' scale factors. Trends between the two values quantify the extent to which record scaling is causing biased structural response. Example results obtained using this method suggest that records selected based on the ground motion parameter ε (or that otherwise account for the spectral shape implied by ε) can be safely scaled without introducing any bias, whereas the records selected using other methods have biased structural responses when scaled.

2.1.2 Introduction

An important consideration at the interface between seismology and earthquake engineering is the selection and processing of earthquake ground motions for use in performing dynamic structural analysis. It is common in practice to select recorded ground motions and 'scale' them by increasing their amplitude to match a desired earthquake intensity level. Due to limitations in recorded ground motion libraries, scaling will continue to be used for the foreseeable future to represent extreme ground motions. But questions remain regarding how the ground motions should be selected, and whether the scaled ground motions are truly representative of ground motions with the given intensity level (where intensity is measured using a parameter such as spectral acceleration at the first mode period of the structure). From an engineer's perspective, the question can be phrased, 'will ground motions scaled to a specified intensity produce the same structural responses as unscaled ground motions naturally at that intensity level?'

Early quantitative investigations into ground motion scaling indicated that a suite of ground motions may be safely scaled to the suite's median spectral acceleration value, at a period *T*, without biasing the median response of a structure having the same first-mode period *T* (Shome et al. 1998, Iervolino and Cornell 2005). But recent work suggests that in some other situations record scaling may induce some bias in structural response (Baker and Cornell 2005b, Luco and Bazzurro 2005). This bias appears to result from the scaled ground motions having inappropriate values of spectral shape or the parameter ε, which is an indirect measure of spectral shape (Baker and Cornell 2005a). This conclusion fits with some intuitive concerns about record scaling: namely, that low intensity ground motions have different frequency content than rare or extreme ground motions. Han and Wen (1994), for example, speculated that "scaling an earthquake to attain a target damage level of different intensity is questionable since scaling a ground motion does not account for variations in ground motion characteristics (e.g., frequency content) which change with intensity."

An important feature of these ground motion scaling studies is that record selection and scaling approaches are evaluated by studying the response of structures subjected to these motions. If it can be verified that scaled ground motions produce structural responses similar to those from unscaled ground motions having the same intensity, then it can be concluded that the given scaling approach is valid. This pragmatic viewpoint has also been taken in a recent paper studying record scaling for geotechnical analysis (Watson-Lamprey and Abrahamson 2006).

2.1.3 Record selection

There are a variety of methods for selecting the suite of ground motions to be used for analysis, and the selection method may have an effect on the bias resulting from scaling the ground motions. Four methods of record selection will be considered below. Two of these methods involve the ground motion parameter ε ('epsilon') so its importance will be summarized briefly, followed by a description of the four record selection methods considered.

2.1.3.1 The ground motion parameter ε, and a predictive model for spectral shape

A common finding of record selection research is that structural responses are dependent upon spectral shape, and that if scaled ground motions have the same spectral shape as the target ground motions, the resulting structural responses from scaled ground motions are statistically similar to responses from unscaled ground motions. Magnitude and distance can affect the spectral shape of records, and the ground motion parameter ε has also been seen to be a useful predictor of spectral shape (Baker and Cornell 2005a, 2006b). The parameter ε is defined as the number of standard deviations between the observed spectral value and the median prediction from an attenuation function. Records with large positive ε values at a given period are typically associated with a peak in the response spectrum at that period, because the ε value indicates an extreme/rare spectral value at that period while other spectral values at other periods are not necessarily so extreme. This tendency of high-ε ground motions to have a peaked spectral shape will be an important consideration in the results below. Baker and Cornell concluded that the effect of ε is at least as great as that of magnitude or distance.

To utilize this finding, however, it is necessary to know the response spectrum associated with ground motions having the target ground motion intensity. The well-known Uniform Hazard Spectrum (UHS) is unappealing for this application, as it is an envelope of spectral values associated with multiple ground motions, rather than a description of a single ground motion. Problems with treating the UHS as the spectrum of a single ground motion have been noted by other researchers (Reiter 1990, Naeim and Lew 1995, Bommer et al. 2000).

A more suitable alternative for this problem is to find the conditional response spectrum of a ground motion, given a level of $Sa(T_1)$ and its associated mean (deaggregation-based) causal magnitude, distance and ε value (Baker and Cornell 2006b). To develop the target spectrum, we first specify the first-mode period of the structure of interest, T_1. The $Sa(T_1)$ value corresponding to a target probability of exceedance at the site is then obtained using PSHA and denoted $Sa(T_1)^*$. We then use deaggregation to find the mean of the magnitude, distance and ε values (denoted $\bar{M}$, $\bar{R}$ and $\bar{\varepsilon}$) that cause the occurrence of $Sa(T_1)^*$ level (e.g., McGuire 1995). $\bar{M}$ and $\bar{R}$, in turn, via ground motion prediction models, determine the means and standard deviations of the response spectral values for all periods, and $\bar{\varepsilon}$ specifies the number of standard deviations away from the mean the ground motion is at the first-mode period, T_1. Given knowledge of the mean ε at T_1, denoted $\bar{\varepsilon}$ (T_1), we can calculate the conditional distribution of Sa values at other periods using only the deaggregation data and knowledge of correlations of ε values at a range of periods, as will be shown below.

This scheme for developing a target spectrum follows from procedures to develop target spectra for analysis of nuclear facilities (DOE 1996, NRC 1997, ASCE 2005), except that those methods incorporate only the causal M and R values from deaggregation. The target spectra must then be scaled up to match the specified Sa value. Here, the effect of ε is incorporated as well, given the finding that ε is a useful predictor of structural response.

The mean target response spectrum based on $\bar{M}$, $\bar{R}$ and $\bar{\varepsilon}$ can be computed in the following manner

$$\mu_{\ln Sa(T)|\ln Sa(T_1)=\ln Sa(T_1)^*} \approx \mu_{\ln Sa}(\bar{M},\bar{R},T) + \sigma_{\ln Sa}(\bar{M},T)\rho_{\ln Sa(T_1),\ln Sa(T)} \cdot \bar{\varepsilon}(T_1) \tag{1}$$

$$\sigma_{\ln Sa(T)|\ln Sa(T_1)=\ln Sa(T_1)^*} \approx \sigma_{\ln Sa}(\bar{M},T)\sqrt{1-\rho^2_{\ln Sa(T_1),\ln Sa(T)}} \tag{2}$$

where $\bar{M}$, $\bar{R}$ and $\bar{\varepsilon}(T_1)$ come from deaggregation given $Sa(T_1) = Sa(T_1)^*$. The terms $\mu_{\ln Sa}(\bar{M},\bar{R},T)$ and $\sigma_{\ln Sa}(\bar{M},Th)$ are the marginal mean and standard deviation of lnSa at T, as predicted by a ground motion prediction ('attenuation') relationship. A model for the correlation term $\rho_{\ln Sa(T_1),\ln Sa(T)}$ is given by Baker and Cornell (2006a). Note that the substitution of mean values for magnitude, distance and ε, rather than the complete deaggregation distributions, is an approximation, but is believed to be accurate in most practical situations (Baker and Cornell 2005b, Appendix E). Examples of this target spectrum are shown in Figure 1a, conditioned on the Sa(0.8s) level exceeded in Los Angeles with probabilities of 50%, 10% and 2% in 50 years. The spectrum given by Equation 1 will be termed a 'Conditional Mean Spectrum, considering ε,' (CMS-ε) because it is a mean value,

conditional on a target $Sa(T_1)$ value, and it considers ε, unlike similar spectra specified by nuclear facility guidelines.

This target spectrum differs from the more commonly used Uniform Hazard Spectrum (UHS) (McGuire 2004). A 10% in 50 years UHS is shown in Figure 1b, along with CMS-ε for three periods of interest. The UHS is presented here to illustrate the relative difference of the proposed conditional mean spectrum, but is not used in the results that follow. Several observations can be made from Figure 1: the CMS-ε is dependent upon the *Sa* value of interest (as seen in Figure 1a, where the spectrum becomes more peaked as $Sa(T_1)$ increases), as well as the period of interest (as seen in Figure 1b). The spectrum also depends upon the site of interest, which will affect the magnitudes and distances to the causal faults.

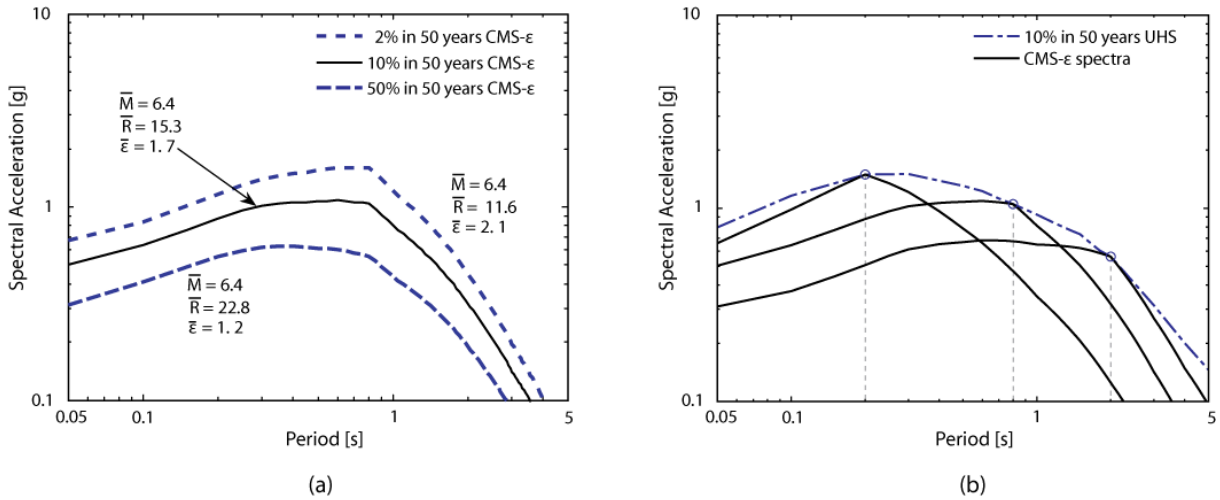

Figure 1: (a) Conditional mean spectra, considering ε, for a site in Los Angeles, given occurrence of *Sa*(0.8s) values exceeded with 2%, 10% and 50% probabilities in 50 years. (b) Uniform hazard spectrum corresponding to 10% probability of exceedance in 50 years, and conditional mean spectra associated with *Sa* values at T_1 = 0.2, 0.8 and 2 seconds.

2.1.4 Record selection methods

When choosing ground motions for estimation of structural response, ideally the distribution of magnitude, distance *and* ε values in the record set would equal the condition distribution of magnitude, distance and ε values seen at the site of interest, given $Sa(T_1)$ (as determined from PSHA deaggregation). Note that this conditional distribution will change as a function of the $Sa(T_1)$ level, so different records would need to be selected for different $Sa(T_1)$ levels. Matching all of these values simultaneously poses practical challenges when selecting from a finite set of recorded ground motions, so it would be helpful to understand which parameters have the greatest effect on the resulting structural response, so that greatest priority can be given to matching those parameters.

To test the relative effect of the *M*, *R* and ε values on structural response, four record-selection methods are now considered, and the resulting structural response outputs compared:

1. Select records at random from a record library, without attempting to match any specific record properties. This will be abbreviated as the 'AR Method,' as it uses Arbitrary Records.
2. Select records with magnitude and distance values representative of the site hazard, without attempting to match the ε values. This will be abbreviated as the 'MR-BR Method,' as it uses *M*, *R*-Based Records.
3. Select records with ε values representative of the site hazard, without attempting to match the magnitude and distance values. This will be abbreviated as the 'ε-BR Method,' as it uses ε-Based Records.
4. Select records with spectral shapes that match the conditional mean spectral shape given by Equation 1, but make no further attempt to directly match the target *M*, *R* or ε values. This will be abbreviated as the 'CMS-ε Method,' as it uses the Conditional Mean Spectrum, considering ε.

For each *Sa* level of interest, 40 ground motions were selected using each of the methods (the specific records selected are listed in Baker and Cornell 2005b). The response spectra of the records selected using Method 4 are shown in Figure 2a, and the mean of the spectra associated with each of the four methods are shown in Figure 2b. In this figure, the period of interest is 0.8 seconds and the target *Sa*(0.8s) value is 0.6g. The $\bar{M}$, $\bar{R}$ and $\bar{\varepsilon}$ associated with the example site and *Sa*(0.8s) value are 6.4, 18 km and 1.5, respectively. In Figure 2a, note that while the individual spectra follow the general shape of the target spectrum and exactly equal the target at 0.8 seconds (due to scaling), there is still variability in the spectra at other periods. In Figure 2b, note that the spectra associated with methods 3 and 4 have a peak at 0.8 seconds, while records selected using methods 1 and 2 do not.

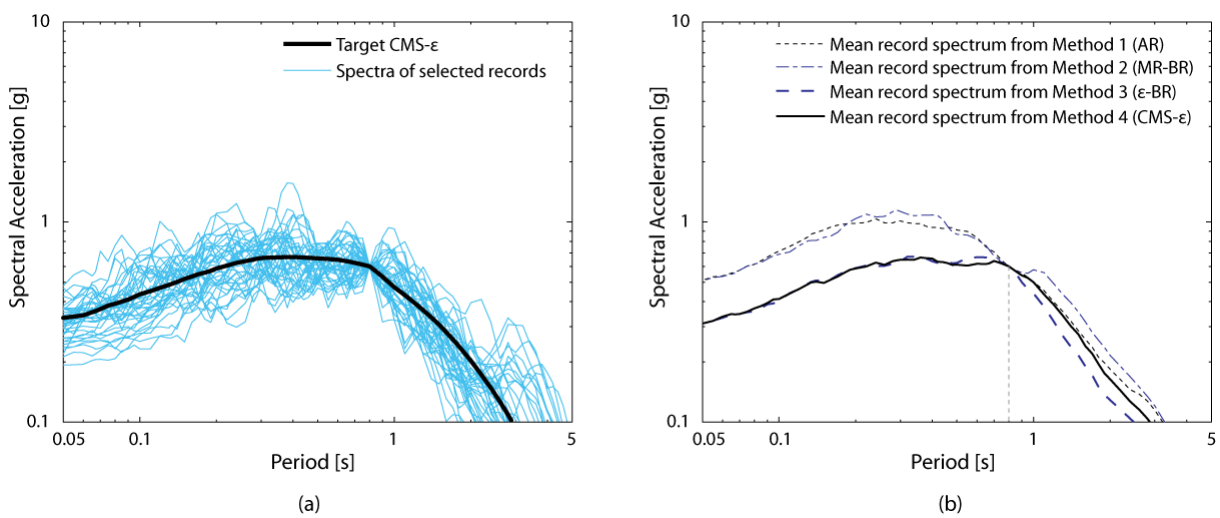

Figure 2: (a) Conditional Mean Spectrum at *Sa*(0.8s)=0.6g (given $\bar{M}$ =6.4, $\bar{R}$ =18 km and $\bar{\varepsilon}$ =1.5) and the response spectra of records selected to match it (i.e., using Method 4). (b) The mean response spectra of record sets selected using each of the four proposed record selection methods, given *Sa*(0.8s)=0.6g.

2.2 Structural model

To demonstrate the proposed evaluation approach, an example analysis was performed using a seven-story reinforced concrete moment frame building. This structure, which was studied as part of a larger research effort (Krawinkler 2004) is located in the Los Angeles area, at the

same site for which the ground motion hazard analysis above was conducted. A 2D model of the transverse frame created by Jalayer (2003) is used here. This model has an elastic first-mode period of 0.8 seconds (which is why *Sa* at 0.8s has been used in the above examples) and uses nonlinear elements with cyclic strength and stiffness degradation in both shear and bending (Pincheira et al. 1999).

2.3 Testing for biased structural response

To detect potential bias from ground motion scaling, we are interested in examining trends between ground motion scale factors and the resulting structural response. Figure 3 presents results from the records selected and scaled to match *Sa*(0.8s) = 0.6g. Each sub-figure shows structural response levels and scale factors associated with one record-selection method. Linear least-squares regression (applied to the logarithms of the variables) is used to estimate the relationship between these two values. If the regression line has a slope of zero, then records with large scale factors are unbiased (i.e., the mean estimated response is independent of record scale factors). A visual inspection of Figure 3 suggests that the record sets selected with the AR Method and the MR-BR Method show some bias, while the record sets selected with the ε-BR Method and the CMS-ε Method show no bias.

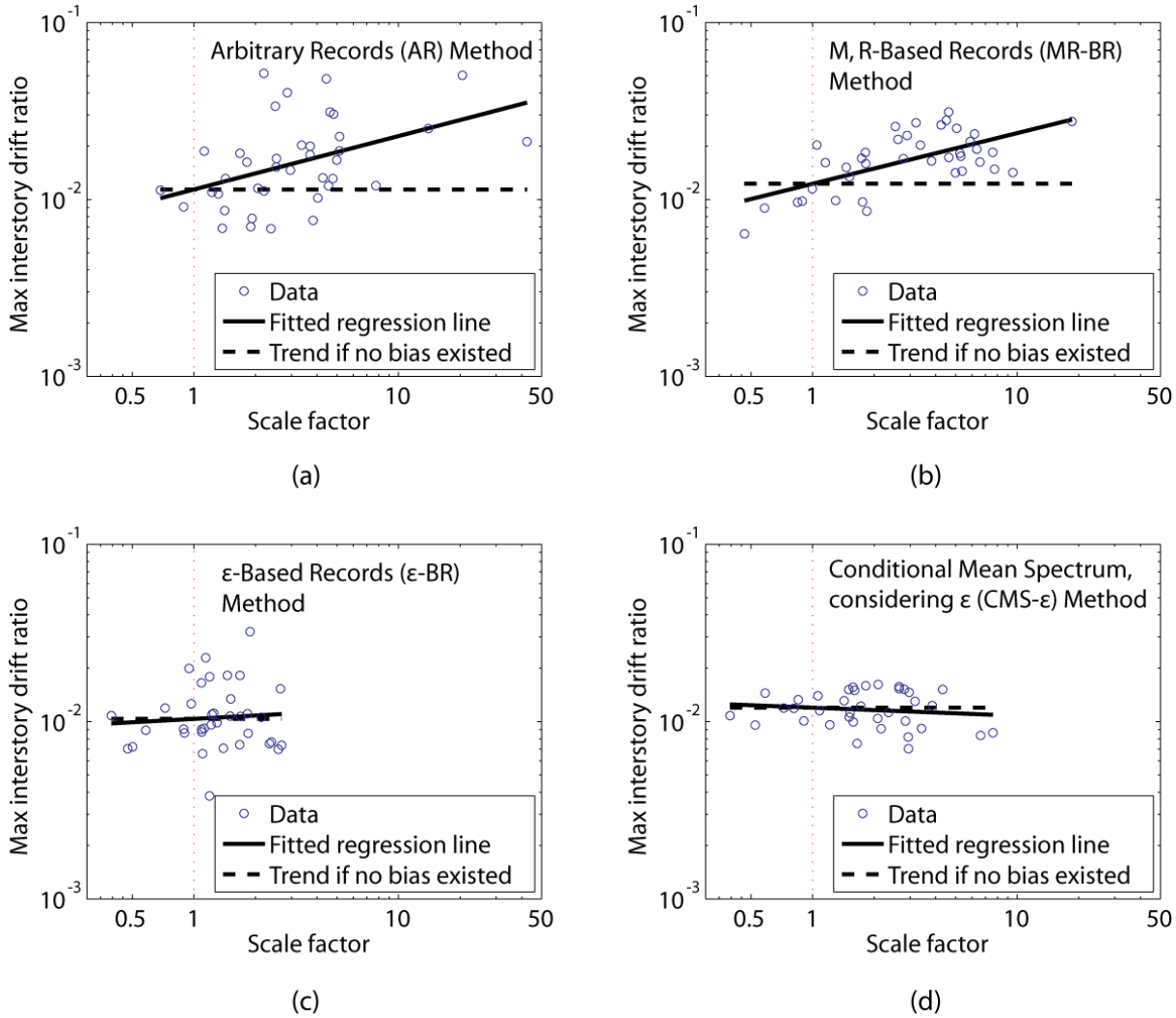

Figure 3: Maximum interstory drift ratio versus record scale factor for each of the four selection methods considered, at an *Sa*(0.8s) level of 0.6g. Regression fits based on scale factor are shown with solid lines. Dashed horizontal lines corresponding to the mean prediction at a scale factor of one are shown for comparison. (a) Records using the AR Method. (b) Records using the MR-BR Method. (c) Records using the ε-BR Method. (d) Records using the CMS-ε Method.

The significance of the slopes from the regression analyses of Figure 3 can be measured using a common statistical diagnostic tool known as an F-test (Kutner et al. 2004). This test produces a probability (referred to as a p-value) that the estimated slope would be as large as, or larger than, the observed slope, given that there was actually no underlying trend in the data (i.e., the probability of erroneously estimating a given slope due to an imprecise estimate from a finite data sample). P-values for the four record-selection methods are reported for six *Sa* levels in Table 1. The row of this table associated with *Sa*(0.8s) = 0.6g provides the p-values for the regressions shown in Figure 3. For this *Sa* level, Methods 1 and 2 have low p-values, indicating that the observed trend is statistically significant. The large p-values for the other two methods indicate that there is likely no underlying trend. When examining all levels of *Sa* in this table, Methods 1 and 2 generally show significant trends with scale factor, while Methods 3 and 4 do not. Similar results were observed when the same test was repeated on

two additional structures (Baker and Cornell 2005b, Appendix F). Note that these slopes could also be used to specify a limit on scale factors, given a maximum allowable bias. It is easier and safer, however, to simply select records using Methods 3 or 4, to completely avoid scaling bias. The possibility also exists that scaling to match intensity measures other than $Sa(T_1)$ could help avoid scaling bias. These alternatives appear preferable to limiting the allowable scale factor while using an approach known to cause bias.

The conclusion here that inappropriate record scaling (from Methods 1 and 2) can bias estimated structural response supports the concern expressed by others that record scaling might fail to modify all ground motion properties in an appropriate way. Through the exploration of conditional mean response spectra above, we have seen that the frequency content of ground motions does in fact change as the intensity (i.e., $Sa(T_1)$) changes. What may be unexpected for readers, however, is that the frequency content is more affected by the variation of ε than by the variation of magnitude or distance. Further, if we select records with the desired spectral shape through a careful record selection scheme (i.e., ε-BR or CMS-ε selection), then we *can* scale records without inducing bias.

Table 1. P-values from regression prediction of max interstory drift ratio as a function of scale factor for four methods of record selection, at six levels of *Sa*(0.8s). P-values of less than 0.05 (indicating statistically significant bias) are marked in boldface.

Sa(0.8s)	Method 1: Arbitrary Records	Method 2: M, R-Based Records	Method 3: ε-Based Records	Method 4: CMS-ε Method
0.1	**0.00**	**0.01**	0.21	0.68
0.2	**0.01**	0.89	0.33	0.07
0.4	**0.01**	0.46	0.73	0.51
0.6	**0.00**	**0.00**	0.67	0.44
0.8	**0.00**	**0.00**	0.73	0.30
1	**0.05**	**0.04**	0.23	0.37
median p-value	**0.01**	**0.02**	0.50	0.40

2.4 Discussion and Conclusions

An approach has been proposed and evaluated for detecting bias in estimated structural response caused by scaling the amplitude of recorded ground motions. The approach involves selecting a suite of ground motions that have been scaled to all have the same intensity level. The suite of ground motions is then used for structural analysis, and the resulting structural responses associated with each ground motion are plotted versus the scale factor associated with that ground motion. Trends between the two values indicate that the record scaling is causing biased structural response (i.e., that scaled ground motions are causing different levels of structural response than unscaled ground motions). In the example presented here, ground motion intensity was measured by the spectral acceleration at the first-model period of the structure, and the structural response parameter of interest was maximum interstory drift ratio. Linear relationships were observed between the logarithm of the records' scale factors and the

associated max interstory drift ratios, so linear least-squares regression on these log values was used to characterize trends.

In order to identify the impact of record selection strategies on potential scaling bias, records were selected using several methods: use arbitrary records, select records to match causal magnitudes and distances, select records to match causal ε values, or select records to match the spectral shape implied by the ground motion's causal magnitude, distance and ε. Causal values of magnitude, distance and ε depend upon the site of interest and the ground motion intensity level of interest, and can be determined from probabilistic seismic hazard deaggregation. A method for calculating this implied spectral shape (given a specified $Sa(T_1)$ level and its associated causal magnitude, distance and ε values) was presented and termed the Conditional Mean Spectrum considering ε (CMS-ε). This CMS-ε is similar to spectra currently used for design of nuclear facilities, except that the recently-identified effect of ε is not considered in those spectra.

It was observed that the presence of scaling bias depended upon the method used to select the ground motions. If the records were selected to account for the peaked spectral shape of 'rare' ground motions (i.e., using the third or fourth method), then the records could be safely scaled up to represent rare (i.e., high $Sa(T_1)$) ground motions while still producing the same structural response values as unscaled ground motions. If records were selected without paying attention to this peaked spectral shape, then scaled-up ground motions produced (on average) larger levels of structural response than unscaled ground motions naturally at the target $Sa(T_1)$ level.

These results may at first glance appear to conflict with some past studies that did not detect scaling bias when record were scaled to target $Sa(T_1)$ values (Shome et al. 1998, Iervolino and Cornell 2005). The reason for the difference is that those two studies were considering a specific problem where the mean scale factor among all the records in a suite was approximately one. In those cases, where as many records were scaled up as were scaled down, the median observed max interstory drift ratio was unbiased (i.e., approximately equal to the median result from the unscaled records). This is consistent with the above results, which predict that biases from scaled-up and scaled-down records would offset, resulting in unbiased median response when the average scale factor is approximately one. Biased responses were observed in another report when mean scale factors were larger than one (Luco and Bazzurro 2005), consistent with the results reported here. This work furthers the result of Luco and Bazzurro by finding that ε-based record selection can overcome the scaling bias that occurs using other record selection methods.

Results are reported here for only one measure of structural response and one measure of ground motion intensity. Ongoing research using the proposed approach will determine the extent to which the conclusions here can be generalized to other structural response measures and ground motion intensity measures. The Pacific Earthquake Engineering Research (PEER) center's Ground Motion and Selection and Modification working group is also evaluating a broader range of record selection techniques than the four described above (http://peer.berkeley.edu/gmsm/). The quantitative test of scaling bias proposed here will thus

aid in providing objective comparisons among the variety of selection and scaling approaches advocated in the scientific literature and professional practice today.

3 Extension of PSHA deaggregation calculations to include deaggregation of ground motion prediction model

3.1 Introduction

Probabilistic seismic hazard analysis (PSHA) deaggregation can be extended to include different ground motion prediction models (GMPM). This section describes why this extension is useful, and how it can be done. To illustrate the extension, an analysis is performed for a simplified site, and used to calculate a conditional mean spectrum (CMS) that incorporates predictions from multiple ground motion prediction models. The proposed approach produces more accurate results than those obtained from simplified methods currently in practice. Deaggregation could be extended even further, to other GMPM predictor variables such as earthquake fault type, to better represent the earthquake events contributing to exceedance of spectral acceleration (*Sa*) values of engineering interest. Potential deaggregation parameters will be identified in Section 3.2.2 and corresponding deaggregation procedures will be described briefly.

3.2 Deaggregation Using Multiple Ground Motion Prediction Models

Computation of conditional mean spectrum requires deaggregation to identify the causal magnitude, distance, and "ε" parameters associated with exceedance of a given ground motion intensity. Since multiple ground motion prediction models are typically used in practice for probabilistic seismic hazard analysis (PSHA), PSHA deaggregation is extended here to include multiple GMPM. To demonstrate, Two methods are used: the former is conditioned on disaggregated means of all ground motion prediction models, and then the CMS is computed by applying equal weights to each model; the latter is conditioned on disaggregated means with respect to each ground motion prediction model, and then the CMS is computed by applying disaggregated weights for each ground motion prediction model.

3.2.1 Ground Motion Prediction Models

Four ground motion prediction models are used in both methods: Abrahamson and Silva (1997), Boore et al. (1997), Campbell (1997), and Sadigh et al. (1997). An equal weight is assigned to each ground motion prediction model, to represent the weighting of these models used by the USGS for the Western US non-extensional tectonic areas (Frankel et al. 2002) [1]. This is done as an initial attempt to use multiple ground motion prediction models, and to serve as a point of comparison to the USGS method.

[1] Campbell (1997) was used in the proposed method instead of Campbell and Bozorgnia (2003) in the USGS method because the former was previously programmed and thus readily available.

3.2.2 Deaggregation Parameters

Different ground motion prediction models have different required input parameters, and this will be important for later deaggregation calculations. Table 2 lists the parameters used in each model. The implications for differences among the input parameters in these models will be discussed in the following sections.

Table 2: Parameters used for four ground motion prediction models

	Abrahamson and Silva	**Boore et al.**	**Campbell**	**Sadigh et al.**
Magnitude	M_0	M_0	M_0	M_0
Distance	r_{rup}	r_{jb}	r_{rup}	r_{rup}
Fault Type	SS, RV, others	RV, RO, others	SS, RV (TR, RO, TO)	SS, RV(TR)
Hanging Wall Modification	Yes	No	Yes	No
Site Condition	Soil, rock	V_{s30}	Soil, soft rock, hard rock	Deep soil, rock

M_0 = Moment Magnitude, SS = strike-slip, RV = reverse, TR = thrust, RO = reverse oblique, TO = thrust oblique

Deaggregation of magnitude is straightforward, since all four models use moment magnitude. Similarly, the ε parameter used in PSHA consistently defined regardless of the ground motion prediction model.

Deaggregation of distance presents one complication, in that the "distance" difference definition is not uniform for all ground motion prediction models. Closest distance to the rupture, r_{rup}, is used by Abrahamson and Silva, Campbell, and Sadigh et al., while Joyner Boore distance, r_{jb}, is used by Boore et al.

Deaggregation of fault type also presents some complications, because the fault types are not defined uniformly among the GMPSMs. A random fault type is assumed in the current USGS deaggregation method. An alternative to the USGS method may use deaggregation based on available fault types in each ground motion prediction model. Fault types can be treated as discrete random variables, sometimes with several lumped into one group. The relative contribution of each fault type can be represented through histograms similar to those for discretized magnitude and distance.

Style of faulting factor is the difference in ground motion between reverse and strike-slip events. Instead of using a constant style of faulting factor that apply to all magnitudes, distances and periods, Abrahamson and Silva uses one that is magnitude and period dependent, Boore et al. use one that is period dependent, while Campbell and Sadigh et al. use one that is magnitude and distance dependent (Abrahamson and Silva 1997). Due to the dependence of style of faulting factor on other parameters, a more rigorous model for

deaggregation of fault types should consider joint distribution of magnitude, distance, period and fault types[2].

Hanging wall terms are included in Abrahamson and Silva and Campbell for thrust and reverse faults (Frankel et al. 2002). Deaggregation of the presence or absence of hanging walls is similar to that of fault types, in the sense that they are modeled as discrete variables, and they are included in only some models but not others.

While site conditions are modeled differently by the various models, this is not a problem in the PSHA/deaggregation framework. That is because the site conditions are not varying with the various potential earthquakes considered in a PSHA calculation: they are fixed for a given site. So whatever site conditions are assumed at the beginning of the PSHA calculation can be re-used for any later calculations of the CMS.

3.2.3 Deaggregation of GMPMs

Deaggregation of ground motion prediction models is similar in concept to deaggregation of events, even though the concept might not be as comfortable at first reading. This deaggregation tells us the relative contributions of each GMPM, given Sa>y. While equal weights are assigned to each GMPM at the beginning of a PSHA calculation, the contribution of each GMPM to predictions of exceeding Sa>y is not equal. This will be important for later CMS computations that incorporate predictions from multiple GMPMs.

3.3 Example site application

To demonstrate the concepts of PSHA deaggregation and CMS computation using multiple GMPM, the CMS is computed using two approaches:

1. based on disaggregated {M, R, ε} means considering all GMPMs, and weight all GMPM predictions equally

2. based on disaggregated {M, R, ε} means that were computed separately for each GMPM, and weight all GMPM predictions according to the probabilities obtained from deaggregation of GMPMs

Four ground motion prediction models are used for Sa prediction. The spectral acceleration period of interest is 1 second. The target Sa value is 0.9g (i.e., we will deaggregate conditioned on Sa(1s)>0/9g). Probability distributions for an arbitrary component Sa (Baker and Cornell 2006c) are considered.

[2] In general, a more rigorous approach for deaggregation of any parameter will involve the joint distribution of all the parameters of interest.

3.3.1 Description of Site and Events

A site is dominated by two earthquake events as shown in Figure 4, with the relevant model parameters listed in Table 3. Event A, with magnitude 6 and distance 10 km from the site, has an annual occurrence rate of 0.01; Event B, with magnitude 8 and distance 25 km from the site, has an annual occurrence rate of 0.002. Both events have strike slip mechanisms, and are vertical ruptures that extent to the ground surface; this simplifies our calculations, because in this case r_{rup}, is the same as r_{jb} and there are no hanging walls (eliminating some of the differences between GMPMs that was discussed earlier). The site condition is soil.

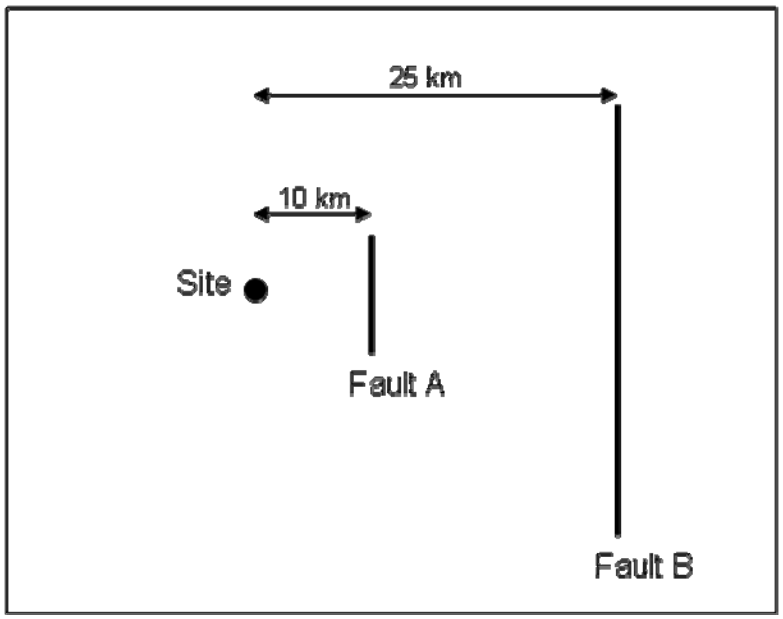

Figure 4: Layout of an example site dominated by two earthquake events A and B.

Table 3: Parameters of Event A and B

	Event A	Event B
Moment magnitude, M	6	8
Distance, R (km)	10	25
Annual rate of occurrence	0.01	0.002
Fault Type	Strike Slip	Strike Slip
Hanging Wall	No	No
Site Condition	Soil	Soil

	A&S	p = 0.25
	Boore et al.	p = 0.25
	Campbell	p = 0.25
	Sadigh	p = 0.25

Figure 5: Logic tree for the example site

The four GMPM discussed above are equally-weighted in the PSHA logic tree, as shown in Figure 5. PSHA calculations are then used to compute the mean annual rate of Sa>y due to 2 events

$$\nu(Sa > y) = \sum_{i=1}^{2}\sum_{j=1}^{4} P(Sa > y \mid Event_i, GMPM_j) P(GMPM_j) \nu(Event_i) \tag{3}$$

Note that this summation is over only the two events, rather than a typical PSHA summation over magnitude and distance, because here there is only a single magnitude and distance associated with each event. The $P(Sa > y \mid Event_i, GMPM_j)$ predictions are shown in Figure 6, and the final result from this PSHA calculation is shown in Figure 7.

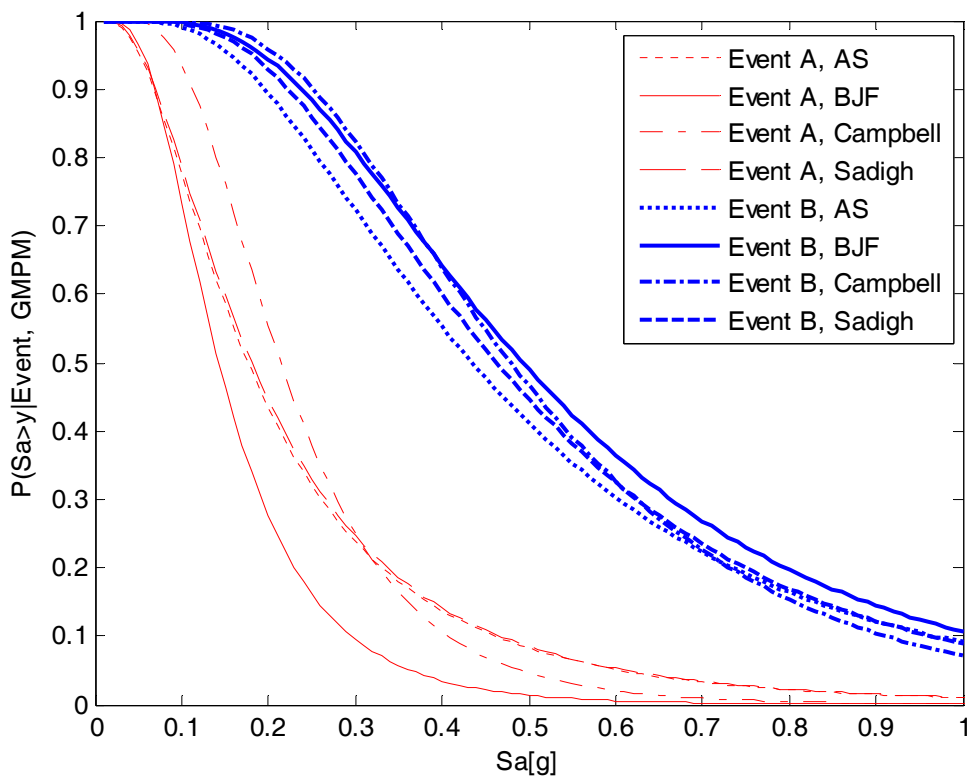

Figure 6: Prediction of ground motion exceeding rate of 2 events using 4 different GMPMs.

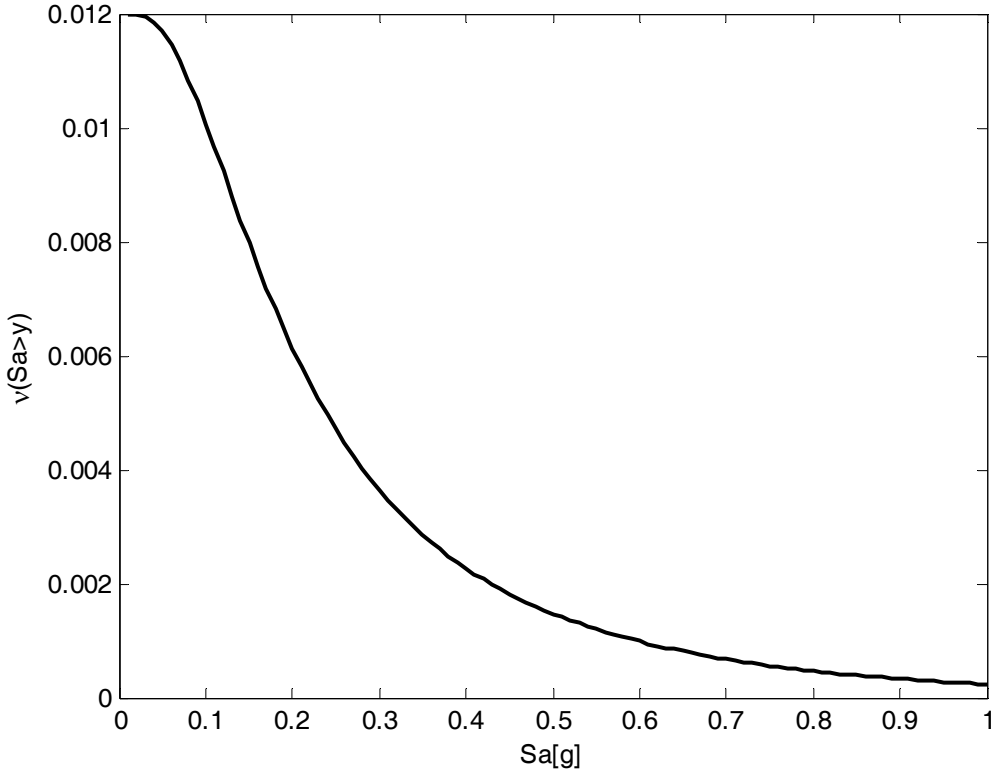

Figure 7: Aggregate earthquake hazard computed using PSHA.

3.3.2 Deaggregation of Events

Deaggregation is now used to find the relative contribution of earthquake events to a site given Sa>y. The conditional distribution of each event is the joint rate of Sa>y and Event i divided by the rate of Sa>y.

$$P(Event_i \mid Sa > y) = \frac{\nu(Sa > y, Event_i)}{\nu(Sa > y)} \tag{4}$$

where

$$\nu(Sa > y, Event_i) = \sum_j P(Sa > y \mid GMPM_j, Event_i) P(GMPM_j) \nu(Event_i) \tag{5}$$

Equation 4 is the same as deaggregation of events using a single GMPM, which is the traditional approach. The only difference is what comes before this, which involves total probability with weighted GMPM (see Equation 5).

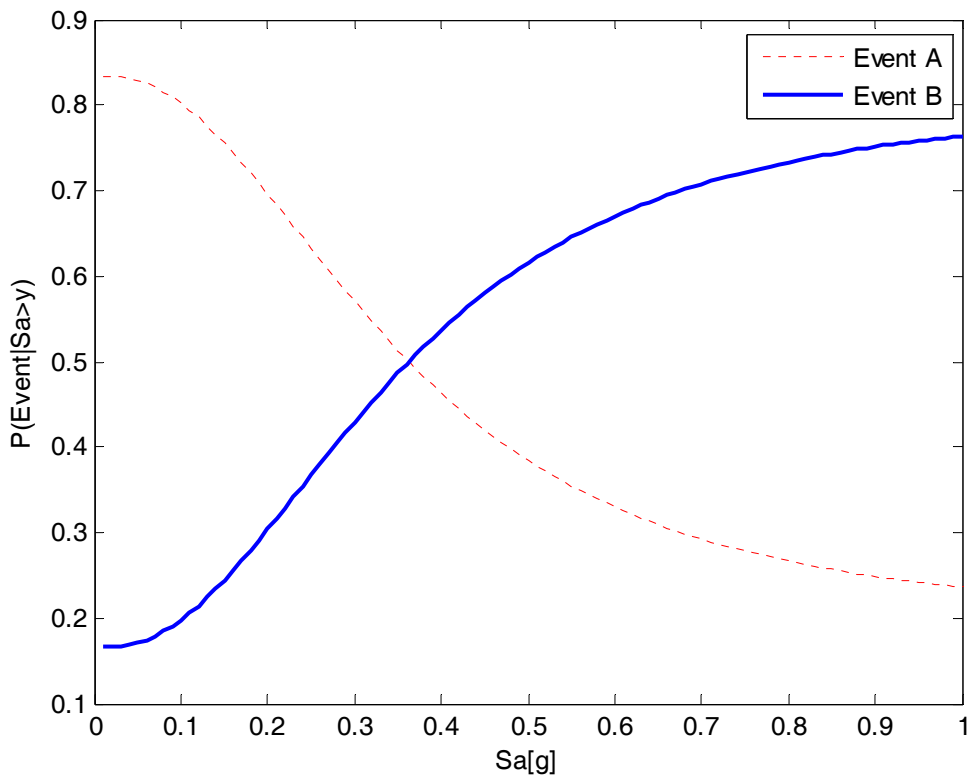

Figure 8: Deaggregation of events using Bayes' Rule

From Figure 8, we see that the smaller but more frequent Event A is most likely to cause exceedance of small Sa amplitudes, but Event B is most likely to cause exceedance of large Sa amplitudes. This is typical of results seen in deaggregation analyses of more complex (i.e., realistic) sites.

3.3.3 Deaggregation of Ground Motion Prediction Models

Deaggregation of GMPM is similar to that of events. Just like deaggregation of events, deaggregation of GMPM will be equal to the joint rate of $Sa>y$ and $GMPM_j$ normalized by total rate of $Sa>y$.

$$P(GMPM_j \mid Sa > y) = \frac{\nu(Sa > y, GMPM_j)}{\nu(Sa > y)} \tag{6}$$

where

$$\nu(Sa > y, GMPM_j) = \sum_i P(Sa > y \mid GMPM_j, Event_i) P(GMPM_j) \nu(Event_i) \tag{7}$$

Note the similarity between Equations 5 and 7.

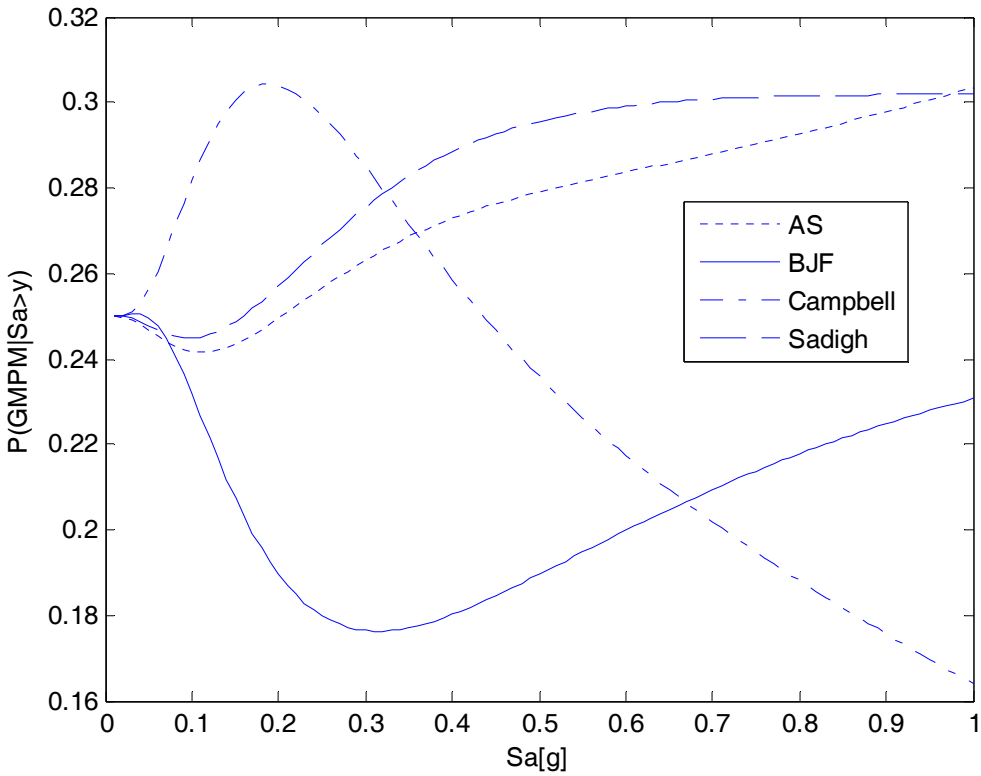

Figure 9: Deaggregation of GMPM, given *Sa* > *y*.

Results from this deaggregation calculation are shown in Figure 9. The deaggregated GMPM contributions vary from 0.16 to 0.31, but practically the difference from their prior weights of 0.25 may not be significant. This figure may be useful on its own, as it offers insights regarding which GMPM contributes most to prediction of exceeding Sa values of interest.

3.3.4 Deaggregation of Magnitude and Distance

Deaggregation of magnitude and distance is very simple for this site, because magnitudes and distances have one-to-one relationships with the causal events considered. Here the mean magnitude associated with Sa > y can be found easily by summing the product of the magnitude given each event and the probability of each event given Sa > y.

$$\bar{M} \mid Sa > y = \sum_i (M \mid Event_i) P(Event_i \mid Sa > y) \tag{8}$$

Disaggregated mean magnitudes for each GMPM can be found using a similar method. The results are shown in Figure 10. In that figure, the thin lines indicate the mean magnitude, given Sa>y *and given that the associated GMPM was the model that predicted Sa>y*. The heavy line provides the mean magnitude, aggregated over all GMPMs.

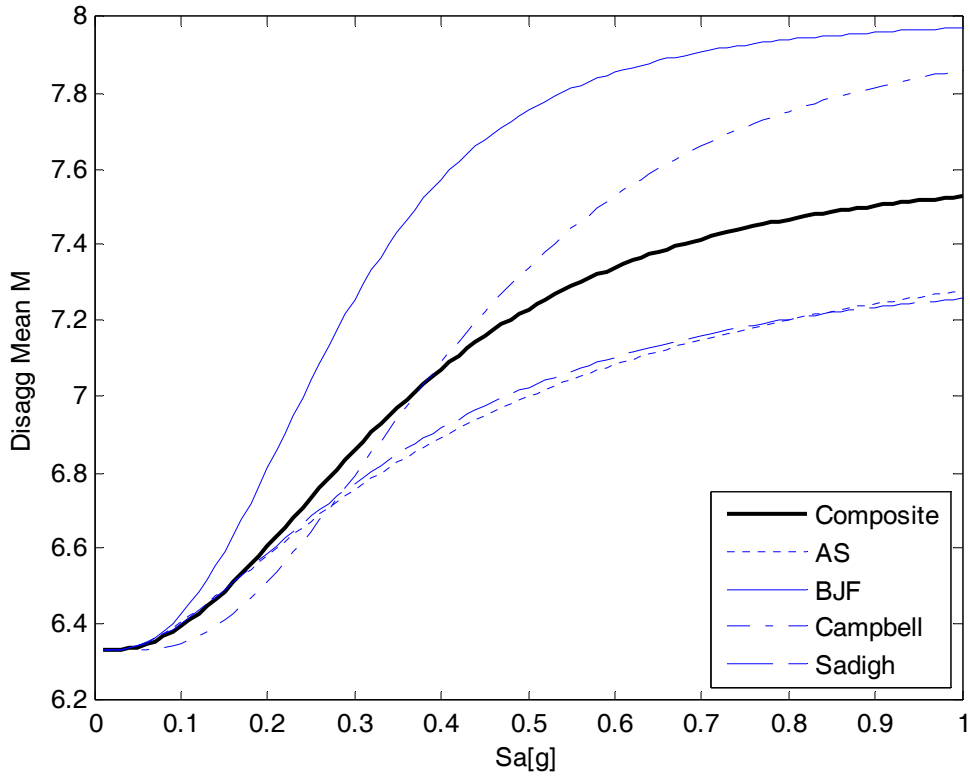

Figure 10: Deaggregation of magnitude M. Lighher

Mean distance deaggregations can be computed using a similar equation

$$\bar{R} \mid Sa > y = \sum_i (R \mid Event_i) P(Event_i \mid Sa > y) \tag{9}$$

This mean, along with the mean distances associated with individual GMPMs, is shown in Figure 11.

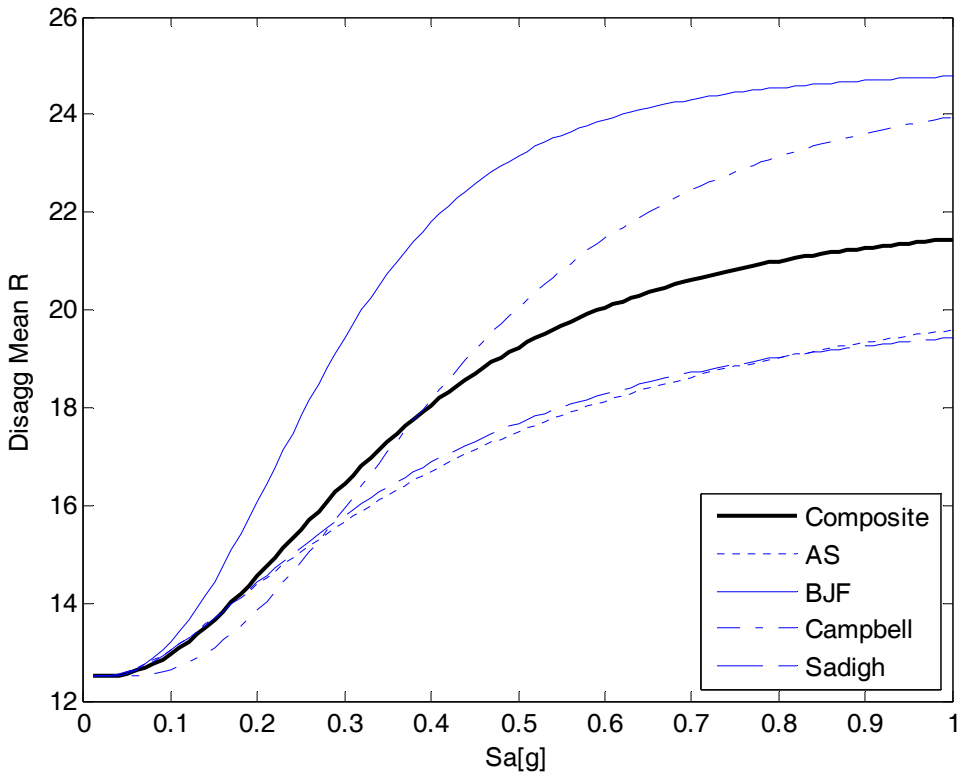

Figure 11: Deaggregation of distance R

3.3.5 Deaggregation of ε

Deaggregation of epsilon is similar to that of magnitude and distance. Note that the epsilon we get first using Equation 10 is the lower bound of epsilon, ε∗. In order to get an expected value of epsilon that causes a certain level of Sa level in Bazzurro and Cornell deaggregation (Sa>y), the centroidal epsilon should be used. This is obtained by simply integrating from disaggregated epsilon to infinity, as shown in Equation 11.

$$\varepsilon^* \mid Sa > y = \sum_i (\varepsilon \mid Event_i) P(Event_i \mid Sa > y) \tag{10}$$

$$\bar{\varepsilon} \mid Sa > y = \int_{-\infty}^{\infty} x f_{\mathrm{E}|Sa>y}(x) dx \tag{11}$$

where

$$f_{\mathrm{E}|Sa>y}(x) = \begin{cases} \dfrac{\phi(x)}{1-\Phi(\varepsilon^*)}, x \geq \varepsilon^* \\ 0, x < \varepsilon^* \end{cases} \tag{12}$$

3.3.6 Conditional Mean Spectrum Computation Using Two Approaches

Conditional mean spectrum can be computed using Equation 1. Note that a slight modification is made to make calculations conditioned on Sa>y rather than Sa=y. Since the target spectral acceleration of interest, Sa(1s), is 0.9g, disaggregated mean values of M, R and ε are obtained, conditioned on Sa(T1=1s)>0.9g. The median predictions of Sa at other periods, using each GMPM, are then conditioned on the deaggregations associated with Sa(1s).

The first approach considered here is to compute the CMS for each GMPM, based on the disaggregated mean magnitude, distance and epsilon values that were computed using all GMPM. The CMS for each GMPM is then weighted equally in an average of the spectra (since the GMPMs were weighted equally in the original PSHA calculation). This is a convenient approach because the overall mean magnitude, distance and epsilon values are readily available from standard deaggregation.

First, we compute the mean M, R, ε given Sa>y using all GMPM.

$$\bar{M} \mid Sa > y = \sum_i (M \mid Event_i) P(Event_i \mid Sa > y) \tag{13}$$

$$\bar{R} \mid Sa > y = \sum_i (R \mid Event_i) P(Event_i \mid Sa > y) \tag{14}$$

$$\varepsilon^* \mid Sa > y = \sum_i (\varepsilon \mid Event_i) P(Event_i \mid Sa > y) \tag{15}$$

The lower bound ε^* is then converted to the centroidal $\bar{\varepsilon}$ as discussed above, to represent the mean epsilon for Sa>y. With this information available, once can compute CMS_j, the CMS computed using $GMPM_j$.

$$CMS_j = CMS_j(\bar{M} \mid Sa > y, \bar{R} \mid Sa > y, \bar{\varepsilon} \mid Sa > y) \tag{16}$$

Finally, we compute a weighted sum of these CMSj, using assigned weight of GMPM.

$$CMS = \sum_j CMS_j P(GMPM_j) \tag{17}$$

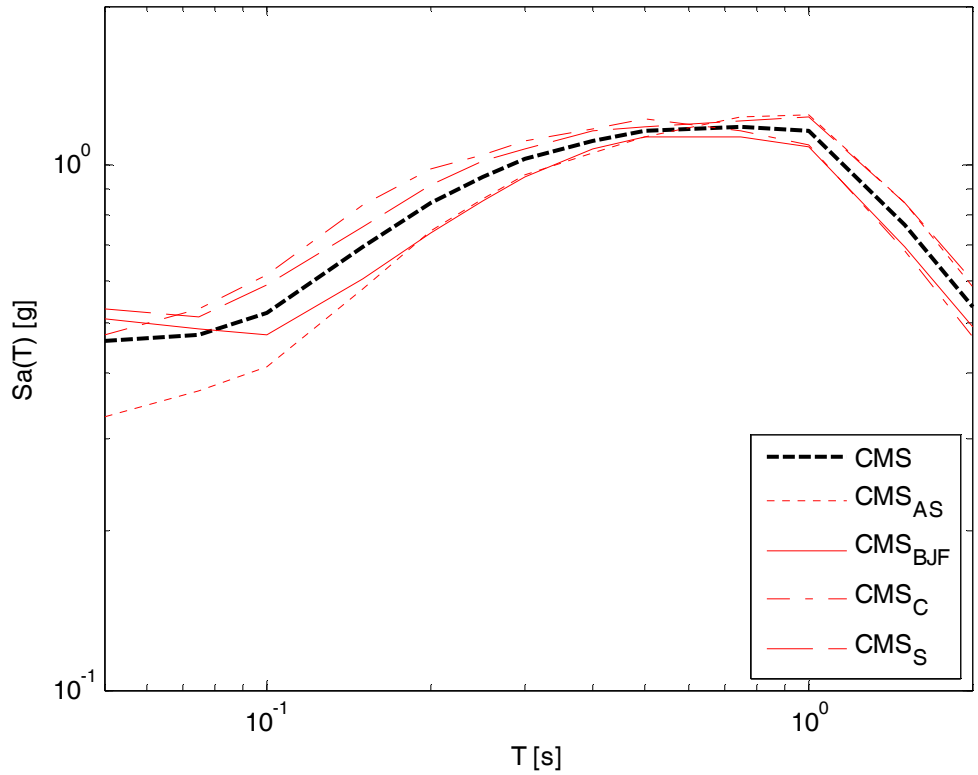

Figure 12: CMS computation using Approach 1

The second approach (and the one that is more probabilistically rigorous) is based on first computing a CMS for each GMPM based on disaggregated mean magnitude, distance and epsilon *obtained for that individual GMPM* given Sa>y. Additionally, the CMS for each GMPM are averaged, but are weighted by the deaggregated contribution of each GMPM to Sa>y.

To execute this approach, first we compute the mean given Sa>y using each GMPM. Note that the mean here is conditional on each GMPM, instead of all GMPM in Approach 1. For example,

$$\bar{M}_j = \bar{M} \mid GMPM_j, Sa > y = \sum_i (M \mid Event_i, GMPM_j) P(Event_i \mid GMPM_j, Sa > y) \quad (18)$$

We next compute CMS_j, the CMS computed using $GMPM_j$ and the respective mean M, R, ε given each GMPM, from procedures similar to Equation **Error! Reference source not found.**.

$$CMS_j = CMS_j(\bar{M}_j, \bar{R}_j, \bar{\varepsilon}_j \mid Sa > y) \quad (19)$$

Finally, we compute a weighted sum of these CMS_j, using the deaggregated contribution of each GMPM to Sa>y.

$$CMS = \sum_j CMS_j P(GMPM_j \mid Sa > y) \tag{20}$$

Although this weighting of the CMS is more complex than the first approach, due to the need to deaggregate on the GMPM, it is more probabilistically consistent with the concept of PSHA and deaggregation.

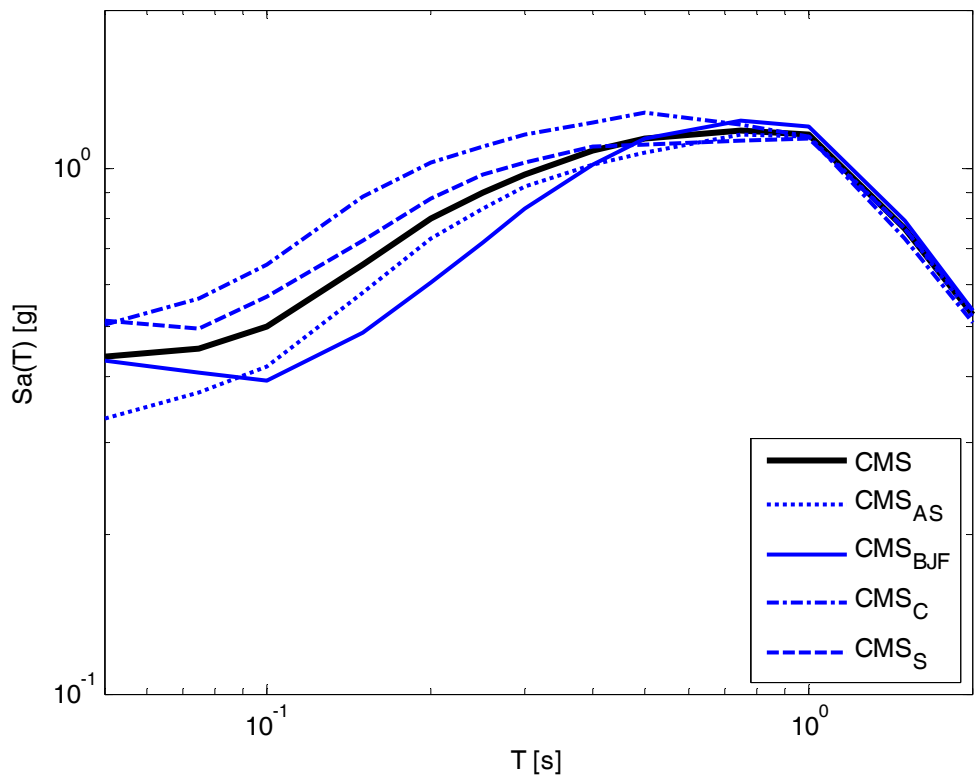

Figure 13: CMS computation using Approach 2

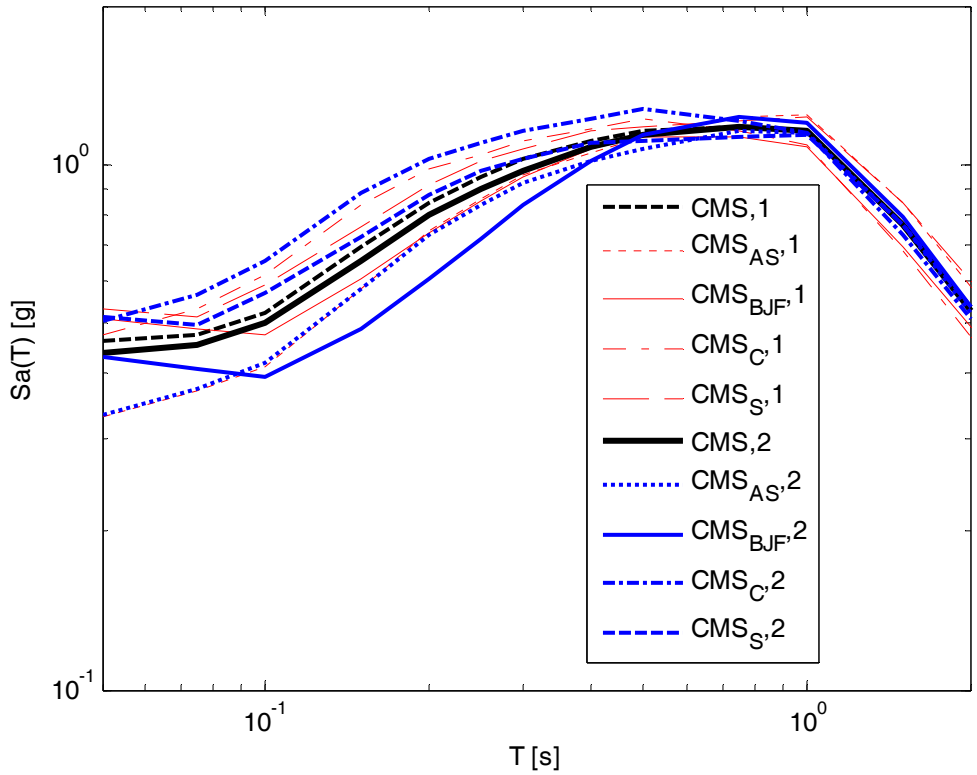

Figure 14: Comparison of CMS computation using Approach 1 and 2

In this example, differences are observed in the CMS spectra produced using the two considered approaches, although the differences are not big. This shows that the two approaches are not equivalent. Differences in the computation of mean magnitudes, distances and ε's, as well as differences in the weighting of the individual CMS spectra, both contribute to the difference. Further work is required to determine whether the observed differences are significant at real sites of interest in practical seismic hazard assessments.

3.4 Conclusions

The computation of the conditional mean spectrum (CMS) using multiple ground motion prediction models (GMPM) requires deaggregation of GMPM. Such an approach is consistent with probabilistic treatment of random variables in traditional probabilistic seismic hazard analysis (PSHA), and is an extension from deaggregation of events that is currently performed in standard PSHA. CMS calculations applied directly to deaggregated mean M, R, ε considering all GMPM, produce different results from calculations applied to disaggregated mean M, R, ε considering each GMPM separately. Differences in the weighting due to GMPM deaggregation can also have an impact on calculations. It is theoretically preferable to consider the conditional relative contribution each GMPM as determined from deaggregation. Deaggregation of GMPMs also provides additional insights regarding which GMPM contributes most to prediction of exceeding Sa values of interest. Although the computation of CMS-ε using multiple GMPM is slightly more complicated, these recently identified

benefits make the calculation worthwhile. Performing GMPM deaggregation in typical PSHA calculations would thus be beneficial in facilitating the improved CMS calculations presented here.

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