NGA West 2
Estimation of Epistemic Uncertainty

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Components of Epistemic Uncertainty

1. Model to model differences
   – Represented by 5 NGA models

2. Uncertainty in predictions from an individual GMPE
   – Can be quantified statistically

3. Other uncertainty not covered by 1 & 2
Model to Model Component

• Measured by computing standard deviation of GMPE predictions as a function of $M$ and $R$

$$\sigma_{\mu_{\ln(\text{SA}|M,R)}} = \sqrt{\frac{\sum_i w_i [\mu_{\ln(\text{SA}|M,R)_i} - \mu_{\ln(\text{SA}|M,R)}]^2}{\sum_i w_i}}$$

with

$$\mu_{\ln(\text{SA}|M,R)} = \frac{\sum_i w_i \mu_{\ln(\text{SA}|M,R)_i}}{\sum_i w_i}$$
Example Calculations

- Set up a set of scenarios for $M$, $R$, and faulting type
- Compute median values for NGA GMPEs, $\mu_{(\ln \text{SA}|M,R)}$
- Computed standard error of these median estimates
Model to Model $\sigma_{\mu}$
Uncertainty in Median Prediction for a Single GMPE

- Simple linear model

\[ y = a + bx \]

- Uncertainty in mean of \( y \) given a new value \( x_0 \)

\[ \sigma^2_{\bar{y}|x_0} = \frac{\sigma^2}{n} \left[ 1 + \frac{(x_0 - \bar{x})}{\sigma_x^2} \right] \]

- GMPE

\[ \sigma^2_{\ln(y)|x_0} = f^T \left[ F^T V^{-1} F \right]^{-1} f \]

\[ F = \left. \frac{\partial \ln(y)}{\partial C} \right|_{x_i} \]

Gradient of model with respect to coefficients \( C \) evaluated at data \( x_i \) used in regression

\[ f = \left. \frac{\partial \ln(y)}{\partial C} \right|_{x_0} \]

Gradient of model with respect to coefficients \( C \) evaluated at new data \( x_0 \) used for prediction

\[ V \]

Block diagonal variance matrix
Example Calculations for Chiou and Youngs (2008)
NGA West 2 Epistemic Model

• Compute variance of model predictions for each of the NGA GMPEs
  – Begin with NGA 2008
  – Repeat with final NGA 2012

• Provide recommended model to represent epistemic uncertainty in individual NGA GMPEs for inclusion in final composite epistemic uncertainty recommendation from developers